Math 117 Lecture #1 notes
The Processes of Mathematical Inquiry:
• Problem Solving
• Reasoning
• Communicating

Industial Age = math literacy meant mastery of basic skills
Information/Technology Age = math literacy means using math to solve new problems

NCTM Standards, Grades 5-8: The math curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can:
• Use problem-solving approaches to investigate and understand math content
• Formulate problems from situations within and outside math
• Develop and apply a variety of strategies to solve problems, with emphasis on multi-step and non-routine problems
• Verify and interpret results with respect to the original problem situation
• Generalize solutions and strategies to new problem situations
• Acquire confidence in using math meaningfully

Many student and teachers equate problem-solving with determining the answers to story or word problems. Many people view word problems as dense, difficult, discouraging, deflating, disturbing, debilitating, disgusting, deadening, demonic, and/or damnable — to mention a few of the more polite adjectives. Asked how she was doing in math, one 1st-grader summarized her dislike for word problems by saying, “I’m doing well, except I don’t like those “problem words!”

What is a problem?
Webster’s New Twentieth Century Unabridged Dictionary suggests two different definitions:
1) “In Math, a problem is anything required to be done …”
2) “A problem is a question …that is perplexing or difficult.”

Exercises: Math educators prefer to call a task for which a person already has a strategy for finding the solution an exercise. Because a way of determining the answer is known, an exercise can be done automatically or even thoughtlessly.

Enigmas: a task a person simple ignores or accepts as unsolvable. Because a person has no interest in finding an answer or is convinced it cannot be found, enigmas typically are not given a second thought and are quickly dismissed.

Problems: the dictionary’s second meaning better captures what mathematicians mean by the term problem: a puzzling task for which a person does not have a readily available solution strategy. Because it is useful to distinguish between puzzles that do and do not motivate interest and action, let’s agree that a problem entails (a) the lack of an obvious way to find a solution and (b) an interest in finding the solution. Problems, then, require a thoughtful analysis and, perhaps, an extended effort.

The Task Confronting Children
The chances of solving genuine problems are affected by three sets of factors:
1) cognitive factors include conceptual knowledge (understanding) and strategies for applying existing knowledge to new situations
2) **affective factors** (interest and confidence) influence children’s disposition to solve problems.

3) **metacognition** includes self-monitoring (considering whether a strategy or solution makes sense).

Effective problem solvers typically have another characteristic: flexibility.

**What is Problem-Solving?**

"Problem-solving is the process of confronting a novel situation, formulating connections between given facts, identifying the goal of the problem, and exploring possible strategies for reaching the goal" (Szetela & Nicol, 1992). It requires the problem solver to coordinate "previous experience, knowledge, and intuition" (Schoenfeld, 1989) in order to solve problems where no direct procedures or paths to the solution have been provided.

George Polya, a late Stanford University mathematician, described the experience of problem solving in his book, *How To Solve It*

"A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive facilities, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery."

George Polya’s 4-steps to problem-solving:
1. Understand the problem
2. Devise a plan (or strategy)
3. Carry out the plan
4. Look back

Heuristics that can be helpful in implementing each of Poly’s four steps of problem-solving:

**Heuristics for Understanding the Problem:**
- State the problem in your own words
- Decide what the unknown is
- Decide what information is needed

**Heuristics for Devising a Plan/Strategy:** Problem-Solving Strategies

- Break into smaller problems
- Change your point of view
  - Example: Mrs. Andrews planted 10 tomato plants in 5 rows of 4 plants each. How did she do this?
- Classify
- Combine strategies
- Do a simulation
- Dramatize the situation
  - Example: There are 8 schools in the Okaw Athletic Conference. How many games must be scheduled for each school’s team to play every other conference team once? Twice? n times?
- Draw a diagram (Venn or other type)
  - Example: Write the letters in the appropriate sections of the Venn diagram using the following:
    - Set A contains the letters in the word IOWA.
    - Set B contains the letters in the word HAWAII.
    - Set C contains the letters in the word OHIO.
    - The Universal set contains the letters in the word WASHINGTON.
Example: A reporter for a high school newspaper interviewed 15 seniors during lunch. He reported that 10 are taking mathematics and physics, 5 are taking physics and chemistry, 7 are taking chemistry and mathematics, and 3 are taking all three subjects. The editor chastised the reporter claiming that the poll was not accurate. Was the editor correct? Why or why not?

Example: The Red Cross looks for three types of antigens in blood tests: A, B and Rh. When the antigen A or B is present, it is listed, but if both of the antigens are missing, the blood type is O. If the Rh antigen is present, the blood is positive, otherwise it is negative. If a lab technician reports the following results for 100 people, how many were classified ) negative? Explain.

Number of samples: Antigen in Blood:

<table>
<thead>
<tr>
<th>Antigen in Blood</th>
<th>Number of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>18</td>
</tr>
<tr>
<td>Rh</td>
<td>82</td>
</tr>
<tr>
<td>A and B</td>
<td>5</td>
</tr>
<tr>
<td>A and Rh</td>
<td>31</td>
</tr>
<tr>
<td>B and Rh</td>
<td>11</td>
</tr>
<tr>
<td>A, B, and Rh</td>
<td>4</td>
</tr>
</tbody>
</table>

Example: Show how it is possible to place 27 beans in 8 cups so that each cup contains a different number of beans?

Eliminate possibilities

Guess and test (or guess and check)

Example: toothpick or matchstick problems

Identify sub-goals

Look for a formula

Look for a pattern

Example: On the first day of math class, Mary filled in a grid with her name as shown:

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M A R
Y M A
R Y M
A R Y
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Write out what row 121 would be. Explain the math used to figure this out.

Make a chart

Make a graph

Make a list

Make a scale drawing

Make a table

Example: Today one blade of grass took root in Clear Pond. Every day the grass population doubles - that i.e., tomorrow there will be two blades of grass, the next day, four blades, and so on. On the 10th day, how many blades of grass are in the pond? If the capacity of the pond is one million blades or grass, on what day will it be filled? Estimate first and then figure it out.

Solve a related problem

Solve a simpler problem

Example: How many squares of different sizes are there on and 8x8 checkerboard?

Solve an equivalent problem

Take a sample

Use a model

Use a variable

Use a variable
Use cases
Use coordinates
Use dimensional analysis
Use direct reasoning
Strategy: Logical Reasoning
Example: Mona, Rita, and Sandra are married to Allen, Fred, and John, but
Sandra does not like John,
Rita is married to John's brother, and
Allan is married to Rita's sister.
Who is married to whom?
Example: magic squares
Use examples
Use indirect reasoning
Use objects
Use properties of numbers
Use properties of numbers
Use symmetry
Work backwards
Example: Shane gave one-half of his rock star pictures to Samantha, then gave 6 pictures to
Daryl and had 12 left. How many rock star pictures did Shane have before he gave any away?
Write an equation
Example: Elaine bought 5 peaches and 3 apples. She figures out that she would have to pay 8 cents
more if she bought 3 peaches and 5 apples. What is the difference between the price of 1 peach
and 1 apple?
Example: A bottle and a cork together cost $1. If the bottle costs 90¢ more than the cork, how
much does the cork cost?
Use a calculator as a problem-solving tool.

Heuristics for Carrying Our the Plan
Children need to recognize the importance of monitoring their efforts and deciding whether or not their
chosen plan is going to get them where they want to go. Genuine problem solving involves discovering a
procedure is completely wrong, discovering a procedure is only part of the answer, and discovering a
procedure may work but only eventually.
• Decide if a new point of view is needed
• Determine whether all relevant information has been used
• Consider whether there are easier ways to solve the problem

Heuristics for Looking Back
Children should consider whether their solution method and answer to the problem at hand might apply to
other problems.
• Decide whether the solution is reasonable
• Decide whether the solution answers the question
• Decide if there are other solutions

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Reasons for Focusing on Communicating
"Both Mathematics and Mathematical learning are, at heart, social activities." (Schoenfeld, 1992).

**NCTM Standards, Grades 5-8:** The study of math should include opportunities to communicate so that students can:
- Model situations using oral, written, concrete, pictorial, graphical, and algebraic methods
- Reflect on and clarify their own thinking about mathematical ideas and situations
- Develop common understandings of mathematical ideas, including the role of definitions
- Use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas
- Discuss mathematical ideas and make conjectures and convincing arguments
- Appreciate the value of mathematical notation and its role in the development of mathematical notations and its role in the development of mathematical ideas.

**Math Proofs and Theorems:** To provide a basis for understanding the role of formal proofs and theorems in high school, and elementary teachers can help students recognize the social nature of knowledge – the need for public verification of ideas. This can probably be best achieved by encouraging them to propose, defend, and evaluate conjectures and proofs themselves.

**Math as a Language:** As an invaluable tool for communicating a variety of ideas clearly, precisely, and succinctly, and serves as the language of engineering and commerce, math has been called "the universal language." Success in a wide variety of fields depends on mastering the "second" language of math, which can be facilitated by actually using it in purposeful efforts to communicate ideas.

**Writing** across the curriculum is gaining popularity among elementary teachers. Unlike other subjects, writing has never been viewed as a natural part of math instruction. NCTM Standards recommends putting more emphasis on expressing math ideas in writing. Traditionally, writing instruction focused on the process of communicating ideas. More recently, writing has been viewed as process of "thinking aloud on paper." Writing to learn movement is fundamentally about using words to acquire concepts; it uses writing to foster reflection and understanding. Writing is a useful tool, forces a child to slow down, be reflective.

Communication activity: "**Design Teaching**"
The aim of Design Teaching is for a teacher to communicate verbally a description of a design so that a student or small group of students can reproduce it. The rules specify that the teacher may verbally describe the design but may not gesture or picture the design. Students may ask questions at any time. The teacher's success can be gauged by whether or not the student reproduced the design. Several elements facilitate communication: precise language, understanding mathematical terms, and relating information to a person's existing knowledge.

Reasoning is an essential tool for math and for everyday life. Learning about logic can provide students with a powerful tool for evaluating their own and other's reasoning.

**NCTM Standards, Grades 5-8:** Reasoning shall permeate the mathematics curriculum so that students can:
- Recognize and apply deductive and inductive reasoning
- Understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs
• Make and evaluate mathematical conjectures and arguments
• Validate their own thinking
• Appreciate the pervasive use and power of reasoning as part of mathematics.

What types of reasoning do mathematicians use when doing mathematics and why should reasoning be a key aspect of the elementary curriculum?

• **Conjectures** = guess, inference, theory, or prediction based on untested or unproven evidence.
• **Empirical tests** = two ways of empirically testing is to search for counterexamples, and to construct a model.
• **Logical tests** = deductive reasoning based on axioms; deductive reasoning in the form of a logical proof is a means for checking and evaluating conjectures.

Looking for patterns and drawing logical conclusions should be integral aspects of elementary math instruction. Conclusions can stem from intuitive, inductive, or deductive reasoning.

• **Intuitive reasoning** involves playing a hunch based on appearances or assumptions.
• **Inductive reasoning** involves looking for a pattern by observing specific examples.
• **Deductive reasoning** entails drawing a logical conclusion, one that necessarily follows from given information. It begins with a premise and leads to a conclusion.

Distinguishing between intuitive, inductive, and deductive reasoning is relatively difficult for many pre-service and in-service teachers because their mathematical training provided little opportunity to reflect on the different types of reasoning and explicitly identify them.

Intuitive reasoning: I bet that white bird in the distance is a swan.

Inductive reasoning: Every swan I have ever seen is white (observation of a pattern among examples) so all swans are white (general rule).

Deductive reasoning: All swans are white (general rule) and the bird being shipped to us is a swan (given); therefore, the bird is white (conclusion about a specific case that necessarily follows from the givens).

*For each of the following, decide the type of reasoning involved.*

Guess my rule.

In-Out Machines.

Am I?

Who Am I Riddles.

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The Goal of lab #1 is to give you problem-solving experience with a variety of problems requiring a variety of strategies and reasoning to find a solution. You may find a calculator helpful for some problems.