Math 117 lecture 12 notes - Probability

Probability theory had its origin in the 16th Century, when an Italian physician and mathematician named Jerome Cardan wrote the first book on the subject, The Book on Games of Chance. For many years the "mathematics of chance" was used primarily to solve gambling problems. It has come a long way since then. Today, the theory of probability is, according to some mathematicians, a "cornerstone of all the sciences." People use probability to predict sales, plan political campaigns, determine insurance premiums and more!
Founders: Blaise Pascal (1623-1662) Pierre de Fermat (1601-1665)

Probabilities are ratios, expressed as fractions, decimals, or percents, determined by considering results our outcomes of experiments.

Experiment = an activity where the results can be observed and recorded
Outcome = each of the possible events of an experiment is an outcome
Sample Space = a set of all possible outcomes for an experiment is a sample space

Classical Rule or Theoretical Rule:

$$P(A) = \frac{\#\text{ of times A occurs in sample space}}{\text{total }\#\text{ of events in sample space}}$$

For any event:
$0 \leq P(A) \leq 1$ *(probability is a fraction or decimal between 0 and 1)*
$P(A) = 0$ means the event canNOT occur
$P(A) = 1$ means the event is certain

Complements: If $P(A) = 5/8$, then $P(\overline{A}) = 1 - 5/8 = 3/8$ (note: $P(A) + P(\overline{A}) = 1$)

Rules of Probability:
"**both-and**" rule:
$P(A \text{ and } B) = P(A) \cdot P(B)$ if independent
$P(A \text{ and } B) = P(A) \cdot P(B|A)$ if dependent
"**either-or**" rule:
$P(A \text{ or } B) = P(A) + P(B)$ if mutually exclusive
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ if not mutually exclusive

Counting Rules:
"Fundamental Counting Rule" = for a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \cdot n$ ways.
"Factorial Rule" = n different items can be arranged in order n! different ways
"Permutation Rule" = the sequences of r items selected from n available items (not allowing repetition) is: $_nP_r = \frac{n!}{(n-r)!}$
""Combination Rule" = the number of combinations of r items selected from n different items is: $_nC_r = \frac{n!}{(n-r)!r!}$

- Pascal's triangle -

A Probability Distribution is a list (distribution) of the outcomes from an experiment along with their respective probabilities.

Experiment: toss 2 coins

Probability Distribution:

Heads	P(H)
0	1/4
1	1/2
2	1/4

The only rules for a probability distribution is that each probability must be a number between 0 and 1, and the sum of all the probabilities must add to 1 (100%).

Ex: Is the following a Probability Distribution?

When four different households are surveyed on Monday night, the number of households with television tuned to Monday Night Football on ABC with their relative frequency is shown (based on data from Nielsen Media Research).

MNF	P(MNF)
0	0.522
1	0.368
2	0.098
3	0.011
4	0.001

The results of an experiment which vary and determined by chance are represented by RANDOM VARIABLES, which may be discrete or continuous. A Probability Distribution is a distribution of all the values of the random variable along with their probabilities. Requirements: P(x) = 1 and $0 \le P(x) \le 1$ for every x

1.

x	P(x)
0	0.25
1	0.25
2	0.25
3	0.25
4	0.25

2.

x	P(x)
5	0.1
10	0.2
15	0.6
13	0.24

3.

x	P(x)
2	0.45
7	0.23
9	1.08

MEAN of a probability distribution (often called expected value) = S [x * P(x)]

Standard Deviation = $\sqrt{\sum(x^2 \bullet P(x)) - \text{mean}^2}$

Ex 1. A commuter airline company finds that for a certain flight, the probabilities of 0, 1, 2, or 3 vacant seats are 0.705, 0.115, 0.090, and 0.090 respectively. Find the MEAN and STANDARD DEVIATION for the number of vacant seats.

Ex 2. 1000 lottery tickets are sold. Each ticket cost $1. Payoff is $900 to a single winner. Find the expected value.

Probability distributions may result from discrete or continuous data. First, we will look at a specific discrete probability distribution, the binomial distribution. Then we will look at a specific continuous probability distribution, the normal distribution.

Probability Distributions

<u>Discrete</u>	<u>Continuous</u>
Binomial	Normal
Geometric	Chi-Square
Poisson	

BINOMIAL EXPERIMENT

Requirements:

1. has a fixed number of trials, n
2. each trial is independent
3. probability of success stays constant for each trial, p
4. outcomes classified into two categories

Notation:

n = number of trials

x = desired number of successes

p = probability of success on single trial

1 – p = probability of failure

Discussion Questions: Which of the following are binomial experiments or can be treated as binomial experiments with negligible error?

1. Testing a sample of five condensers (with replacement) from a population of 20 condensers, of which 40% are defective?
2. Polling 500 voters on the Presidential election from a population of 200,000 voters if 35% are Republican and 65% are Democratic.
3. Firing 20 missiles at a target with a hit rate of 90%.
4. Testing a sample of eight drug dosages from a population of 5000, of which 2% are contaminated.
5. Polling 1000 voters in the presidential election from a population of 8 million voters, of which 40% are Democrats, 35% are Republicans, and 25% are Independent.
6. Tossing an unbiased coin 500 times.
7. Tossing a biased coin 500 times.
8. Surveying 500 consumers to find the brands of toothpaste preferred.
9. Surveying 500 consumers to determine whether their preferred brand of toothpaste is Brand X.
10. Administering a driving test to 50 license applicants with a passing rate of 72%.

Binomial Experiment: *(directions will be given in lecture)*

Binomial formula: $P(x) = \frac{n!}{x!(n-x)!} \cdot p^{x} \cdot (1-p)^{n-x}$

but the TI-83 calculator can be used instead:
DISTR -> binompdf(n,p,x) *or* binomcdf(n,p,x) *when appropriate*

by definition (for binomial): mean = $n \cdot p$
standard deviation = $\sqrt{np(1-p)}$

Example 1: Rates of on-time flights for commercial jets are tracked by the U.S. Department of Transportation. Recently, Southwest Air had the best rate with 80% of its flights arriving on time. A test is conducted by randomly selecting 15 Southwest flights and observing whether they arrive on time. Find the probability that:

- exactly 10 flights arrive on time.
- at least 10 flights arrive on time.
- less than 10 flights arrive on time.

  Would it be unusual for Southwest to have 5 flights arrive late?

Example 2: Nine percent of men (and 0.25% of women) cannot distinguish between the colors of red and green. This is the type of color blindness that causes problems with traffic signals. If six men are randomly selected for a study of traffic signal perceptions, find the probability that:

1. exactly two of them cannot distinguish between red and green.
2. more than two of them cannot distinguish between red and green.

NORMAL DISTRIBUTION
Characteristics:

1. Continuous data (or data treated as continuous)
2. Symmetric
3. Virtual range (99.8% of data) within 3 standard deviations each side of mean (a spread of 6 st. dev.)
4. Total area under the curve = 100%
5. Probability = area under curve between 2 data values

Notation: $\bar{x}$ = mean
s = standard deviation

TI-83 calculator strokes: DISTR ->normalcdf (lower, upper, $\bar{x}$, s)

Example 1:

Heights of women are normally distributed with a mean of 63.6 in. and a standard deviation of 2.5 in. (*based on information from the National Health Survey*). The U.S. Army requires women's heights to be between 58 in. and 80 in. Find the percentage of women meeting that height requirement. Are many women being denied the opportunity to join the Army because they are too short or too tall?

Example 2:

The scores for males on the math portion of the SAT test are normally distributed with a mean of 531 and a standard deviation of *114 (based on information from the College Board*). If the College of Newport includes a minimum score of 600 among its requirements, what percentage of males do not satisfy that requirement?

Example3:

Cans of regular Coke are labeled as containing 12 oz. The contents are normally distributed with a mean of 12.19 oz. and a standard deviation of 0.11 oz. *(based on information from the Coca-Cola Co.*). What percentage of cans contain less than the 12 oz. Printed on the label? Are many consumers being cheated?

Summary of Calculator Strokes for Probability Distributions:

2nd DISTR - >

normalcdf(lower, upper, mean, stan dev) gives the probability between two values of a normal distribution

binompdf(n, p, s) gives the probability for one value of a binomial distribution

binomcdf(n, p, x) gives the cumulative probability for all values from the lowest to and including the given x

MATH - > PRB

RandInt(lower, upper) gives a random digit between “lower number” and “upper number”

nPr evaluates permutation of n things taken r at a time

nCr evaluates combination of n things taken r at a time