Probability theory had its origin in the 16th Century, when an Italian physician and mathematician named Jerome Cardan wrote the first book on the subject, *The Book on Games of Chance*. For many years the "mathematics of chance" was used primarily to solve gambling problems. It has come a long way since then. Today, the theory of probability is, according to some mathematicians, a "cornerstone of all the sciences." People use probability to predict sales, plan political campaigns, determine insurance premiums and much more!

Founders:  Blaise Pascal (1623-1662)  Pierre de Fermat (1601-1665)

Probabilities are ratios, expressed as fractions, decimals, or percents, determined by considering results our outcomes of experiments.

Experiment = an activity where the results can be observed and recorded

Outcome = each of the possible events of an experiment is an outcome

Sample Space = the set of all possible outcomes for an experiment is a sample space

Event = a subset of a sample space

Classical Rule or Theoretical Rule:

\[
P(A) = \frac{\text{# of times } A \text{ occurs in sample space}}{\text{total # of events in sample space}}
\]

Empirical Rule or Experimental Rule:

\[
P(A) = \frac{\text{# of times } A \text{ is observed}}{\text{total # of trials}}
\]

Subjective Rule:

\[
P(A) = \frac{\text{# of times } A \text{ is believed to occur}}{\text{total # of trials}}
\]

For any event:

\[0 \leq P(A) \leq 1\]  (*probability is a fraction or decimal between 0 and 1*)

\[P(A) = 0\] means the event can NOT occur

\[P(A) = 1\] means the event is certain

Complements:

If \( P(A) = \frac{5}{8} \)

then

\[ P(A) = 1 - \frac{5}{8} = \frac{3}{8} \]

\[ P(A) + P(A) = 1 \]
Counting Rules: (purpose to determine the sample space)
"Fundamental Counting Rule" = for a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of m•n ways.
"Factorial Rule" = n different items can be arranged in order n! different ways

"Permutation Rule" = the sequences of r items selected from n available items (not allowing repetition) is: \( nPr = \frac{n!}{(n-r)!} \)

"Combination Rule" = the number of combinations of r items selected from n different items is: \( nCr = \frac{n!}{(n-r)!r!} \)

Example problem for counting rules:

Rules of Probability:
"both-and" rule:
\[
P(A \text{ and } B) = P(A) \cdot P(B) \quad \text{if independent}
\]
\[
P(A \text{ and } B) = P(A) \cdot P(B \mid A) \quad \text{if dependent}
\]

"either-or" rule:
\[
P(A \text{ or } B) = P(A) + P(B) \quad \text{if mutually exclusive}
\]
\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad \text{if not mutually exclusive}
\]

Multi-Stage Events & Tree Diagrams
*The probability of the outcome along any path is equal to the product of all probabilities along the path.*

Socks in the Dark: You have a drawer filled with single socks. If they were mated there would be 2 pair of white socks, 3 pair of tan socks, and 5 pair of black socks. Draw a tree diagram that shows all of the possible ways of picking two socks from the drawer in the dark one sock at a time, without replacement. Then determine the probability that you pick a pair of:
a) black socks
b) tan socks
c) the same colored socks
Classic Probability Problems to think about:
1. Suppose a couple wants 4 children. Which is more likely, 3 of one sex and 1 of another, or 2 and 2 of each sex?
2. If each parent has two genes for blue eyes, what is the probability that their child will have blue eyes?
3. A man says, out of my two children, one is a boy. What is the probability the other is a boy?
In the US, the rate of twins is 1/100.
For triplets, 1/100^2. What is the rate for quadruplets?

Popular probability problems:
• Birthday Paradox (handout)
• Monty’s Dilema (you explore this in the spinners lab)
• Cereal Box Problem (you explore this in the spinners lab)
• Small World Problem:
  Every person is in direct touch with a certain number of people. These are links for chains of acquaintances. Pick any 2 people at random in US. Chances are 1/200000 they know each other, yet probability is better than 1/2 one knows someone who knows someone who knows the other. This "acquaintance" network explains the speed of jokes and gossip around the US.
• Mozart’s Melody Dicer was written as a dice game. The player has the opportunity to compose a waltz consisting of 16 bars, but the exact composition of each bar is determined by the roll of the dice. Mozart composed 11 different options for each of 14 of the bars, 2 options for one bar, and the final bar was fixed. There are, therefore, 2x11^{14} (over 750) variations on this waltz, most have never been heard!

Real-world applications of probability:
Risk is based on analyzing the probability that a negative event – the risk – will occur.
Human intuition is not very accurate when assessing risk. Perceived risk – what people think will happen – often varies greatly from actual risk.
Is it more dangerous to fly on a major US airline or to drive in a car?
Situations controlled by the risk-taker and are voluntary are often perceived to be less risky. Situations that occur less frequently receive more attention when they do; botulism is unfamiliar, cancer is common.
Search and Rescue
Queueing Theory
Weather Forecasting
Car and Life Insurance
Lotteries and Gambling
Is there a real-life parallel to the TV show “NUMBERS”?
YOU DECIDE: Is Probability involved in the solution?
1. You buy two identical notebooks at different stores. One notebook cost $1.89, the other $.99. You started out with $11.00. How much money do you have left?
2. You set our VCR to record a TV show at 7:30 p.m. on Channel 2. You come home at 9:30 pm. Do you know what is recorded on the tape?
3. Your new computer has just arrived. You open the box, set up the computer, and flip the switch to "on." Will it work?

Moral: If outcomes are uncertain -> it’s probability

The Pesky Magician
A pesky Magician comes to the fair each year. He has card tricks and dice games and enjoys surprising people so that they are in awe of his powers. He seems only slightly sinister . . . almost likable . . . and it is unnerving that he uses dice and cards and things you thought you understood, but he always seems to win! You want to know more. You think there must be some trick to what he is doing, because in all your years, you’ve never seen someone repeatedly win at a game, unless there was a trick or it was “rigged.” You watch him carefully . . . you’re planning to challenge him soon! You’ve thought up a good game, so you approach the Magician and say:

"I have a new game I would like to propose to you. Let’s roll 2 dice and ADD them. If the sum is even, you win. If the sum is odd, I win. Your present him with a list of the possible sums: 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11 - 12

But now that you see the list of possibilities, you panic! "Oh no! What did I get myself into?" you ask. Of course, the Magician likes what he sees because he seems to have the advantage. He thinks . . . hmmm . . . 6 are even and only 5 are odd. It looks like an even answer has the advantage . . . true?

The pesky Magician proposes another game to play. He says: "Here is a new game you’ll like. You will roll two dice. If we subtract the smaller amount from the larger amount, the possible answers are: 0 - 1 - 2 - 3 - 4 - 5
I’ll take the answers 0 - 1 - 2 and you take the answers 4 - 5 - 6. OK? We tally the results. After 10 rolls, we see who wins.

The Magician likes to suggest games that are in his favor. You tell him that you are not interested in playing that game, because it is strongly in his favor. "Oh, but I really want to play a game with you," he says. "How about if I can show you a diagram for a game so you can see all the possible combinations? If I can do that, will you play?"
So the Magician explains, "In this game, you roll two dice and multiply them. If the answer is an even number, I get a point. If the answer is an odd number, you get a point. And I'm even going to show you a diagram of the combinations . . ."

Possible combinations for the products of 2 dice:

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Whoa! . . . hold the phone! . . . the pesky Magician watched me diagram the possible outcomes. He saw my face when I discovered the game is NOT fair, so he is very surprised when I look him in the eye and say: "OK, Mr. Magician, I agree to play this game on one condition . . ."

He is so startled to hear me say I will play that he says, "you will? I mean, oh, good. What is the condition?"

"Every time there is an even number answer, you get one point. Every time there is an odd number answer, I get three points. This will adjust the game to make it fair. We will play until the first one scores 21 points. Agree?"

To make an informed decision to play, review the rules, picture the possibilities, regard the risks, and state your chance of success and your chances of losing. This will help you decide what you are willing to risk when playing a game.

A game is FAIR only when it has equally likes outcomes of winning and loosing.