11.1 Exercises

1–10 • Find the first four terms and the 100th term of the sequence.

1. $a_n = n + 1$
2. $a_n = 2n + 3$
3. $a_n = \frac{1}{n + 1}$
4. $a_n = n^2 + 1$
5. $a_n = \frac{(-1)^n}{n^2}$
6. $a_n = \frac{1}{n^3}$
7. $a_n = 1 + (-1)^n$
8. $a_n = (-1)^{n+1} \cdot \frac{n}{n + 1}$
9. $a_n = n^8$
10. $a_n = 3$

11–16 • Find the first five terms of the given recursively defined sequence.

11. $a_n = 2(a_{n-1} - 2)$ and $a_1 = 3$
12. $a_n = \frac{a_{n-1}}{2}$ and $a_1 = -8$
13. $a_n = 2a_{n-1} + 1$ and $a_1 = 1$
14. $a_n = \frac{1}{1 + a_{n-1}}$ and $a_1 = 1$
15. $a_n = a_{n-1} + a_{n-2}$ and $a_1 = 1, a_2 = 2$
16. $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ and $a_1 = a_2 = a_3 = 1$

18. $a_n = \log \left( \frac{n}{n + 1} \right)$

19–46 • Find the

19. $\sum_{k=1}^{n} k$
20. $\sum_{k=1}^{n} \frac{1}{k}$
21. $\sum_{k=1}^{n} [1 + (-1)^k]$
22. $\sum_{k=1}^{n} 2^{k-1}$
Use a graphing calculator to do the following.

10. Find the 10th term of the sequence.
11. Graph the first 10 terms of the sequence.

12. \( a_n = 4n + 3 \)
13. \( a_n = n^2 + n \)
14. \( a_n = \frac{12}{n} \)
15. \( a_n = \frac{1}{a_{n-1}} \) and \( a_1 = 2 \)
16. \( a_n = a_{n-1} - a_{n-2} \) and \( a_1 = 1, a_2 = 3 \)

Find the \( n \)th term of a sequence whose first several terms are given.

17. 2, 4, 8, 16, …
18. \(-\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, \ldots\)
19. 1, 4, 7, 10, …
20. 5, -25, 125, -625, …
21. \( \frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \ldots \)
22. 0, 2, 0, 2, 0, …
23. \( \frac{3}{4}, \frac{5}{4}, \frac{7}{5}, \frac{9}{7}, \ldots \)
24. \( \frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \frac{7}{6}, \ldots \)
25. \( 1, \frac{2}{3}, 3, \frac{4}{3}, \ldots \)

Find the first six partial sums \( S_1, S_2, S_3, S_4, S_5, S_6 \) of the sequence.

26. 1, 3, 5, 7, …
27. \( \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \ldots \)
28. -1, 1, -1, 1, …
29. \( \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots \)
30. \( \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots \)

Find the first four partial sums and the \( n \)th partial sum of the sequence \( a_n \).

31. \( a_n = \frac{2}{3^n} \)
32. \( a_n = \frac{1}{n(n+1)} \)
33. \( a_n = \sqrt{n} - \sqrt{n+1} \)
34. \( a_n = \log\left(\frac{n}{n+1}\right) \) [Hint: Use a property of logarithms to write the \( n \)th term as a difference.]
35. \( a_n = \frac{1}{2^n} \)
36. \( a_n = \frac{1}{n+1} - \frac{1}{n+2} \)
37. \( a_n = \frac{1}{n+1} \)
38. \( a_n = \sqrt{n} + \sqrt{n+1} \)
39. \( a_n = \log\left(\frac{n}{n+1}\right) \) [Hint: Use a property of logarithms to write the \( n \)th term as a difference.]

Find the sum.

40. \( \sum_{k=1}^{10} k^2 \)
41. \( \sum_{k=1}^{5} \frac{1}{k} \)
42. \( \sum_{i=1}^{100} (-1)^i \)
43. \( \sum_{i=1}^{10} [1 + (-1)^i] \)
44. \( \sum_{i=1}^{10} 10 \)
45. \( \sum_{i=1}^{5} 2^i \)
46. \( \sum_{i=1}^{3} i^3 \)
47. \( \sum_{i=1}^{10} k^2 \)
48. \( \sum_{i=1}^{100} (3k + 4) \)
49. \( \sum_{j=1}^{20} j^2(1 + j) \)
50. \( \sum_{j=1}^{15} \frac{1}{j^2 + 1} \)
51. \( \sum_{n=0}^{25} (-1)^{2n} \)
52. \( \sum_{n=1}^{100} \frac{(-1)^n}{n} \)

Write the sum without using sigma notation.

53. \( \sum_{k=1}^{5} \sqrt{k} \)
54. \( \sum_{i=1}^{4} \frac{2i - 1}{2i + 1} \)
55. \( \sum_{k=0}^{5} \sqrt{k + 4} \)
56. \( \sum_{k=0}^{9} k(k + 3) \)
57. \( \sum_{i=1}^{100} i^3 \)
58. \( \sum_{i=1}^{100} (-1)^{i+1} i^3 \)

Write the sum using sigma notation.

59. \( 1 + 2 + 3 + 4 + \cdots + 100 \)
60. \( 2 + 4 + 6 + \cdots + 20 \)
61. \( 1^2 + 2^2 + 3^2 + \cdots + 10^2 \)
62. \( \frac{1}{3} \ln 3 - \frac{1}{4} \ln 4 - \frac{1}{5} \ln 5 + \cdots + \frac{1}{100} \ln 100 \)
63. \( \frac{1 - 1}{1 + 1} + \frac{1}{3} + \frac{1}{3} + \cdots + \frac{1}{999} \cdot 1000 \)
64. \( \sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{10} \)
65. \( 1 + x + x^2 + x^3 + \cdots + x^{100} \)
66. \( 1 - 2x + 3x^2 - 4x^3 + \cdots - 100x^{99} \)
67. Find a formula for the \( n \)th term of the sequence \( \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}, \ldots \)
[Hint: Write each term as a power of 2.]

68. Define the sequence
\[ G_n = \frac{1}{\sqrt[3]{3}} \left( (1 + \sqrt{3})^n - (1 - \sqrt{3})^n \right) \]

Use the TABLE command on a graphing calculator to find the first 10 terms of this sequence. Compare to the Fibonacci sequence \( F_n \).

Applications

69. Compound Interest Julio deposits $2000 in a savings account that pays 2.4% interest per year compounded
monthly. The amount in the account after \( n \) months is given by the sequence
\[
A_n = 2000 \left( 1 + \frac{0.024}{12} \right)^n
\]
(a) Find the first six terms of the sequence.
(b) Find the amount in the account after 3 years.

70. **Compound Interest** Helen deposits $100 at the end of each month into an account that pays 6% interest per year compounded monthly. The amount of interest she has accumulated after \( n \) months is given by the sequence
\[
I_n = 100 \left( 1.005^n - 1 \right)
\]
(a) Find the first six terms of the sequence.
(b) Find the interest she has accumulated after 5 years.

71. **Population of a City** A city was incorporated in 2004 with a population of 35,000. It is expected that the population will increase at a rate of 2% per year. The population \( n \) years after 2004 is given by the sequence
\[
P_n = 35,000(1.02)^n
\]
(a) Find the first five terms of the sequence.
(b) Find the population in 2014.

72. **Paying off a Debt** Margarita borrows $10,000 from her uncle and agrees to repay it in monthly installments of $200. Her uncle charges 0.5% interest per month on the balance.
(a) Show that her balance \( A_n \) in the \( n \)th month is given recursively by \( A_0 = 10,000 \) and
\[
A_n = 1.005A_{n-1} - 200
\]
(b) Find her balance after six months.

73. **Fish Farming** A fish farmer has 5000 catfish in his pond. The number of catfish increases by 8% per month, and the farmer harvests 300 catfish per month.
(a) Show that the catfish population \( P_n \) after \( n \) months is given recursively by \( P_0 = 5000 \) and
\[
P_n = 1.08P_{n-1} - 300
\]
(b) How many fish are in the pond after 12 months?

74. **Price of a House** The median price of a house in Orange County increases by about 6% per year. In 2002 the median price was $240,000. Let \( P_n \) be the median price \( n \) years after 2002.
(a) Find a formula for the sequence \( P_n \).
(b) Find the expected median price in 2010.

75. **Salary Increases** A newly hired salesman is promised a beginning salary of $30,000 a year with a $2000 raise every year. Let \( S_n \) be his salary in his \( n \)th year of employment.
(a) Find a recursive definition of \( S_n \).
(b) Find his salary in his fifth year of employment.

76. **Concentration of a Solution** A biologist is trying to find the optimal salt concentration for the growth of a certain species of mollusk. She begins with a brine solution that has 4.1 g/L of salt and increases the concentration by 10% every day. Let \( C_0 \) denote the initial concentration and \( C_n \) the concentration after \( n \) days.
(a) Find a recursive definition of \( C_n \).
(b) Find the salt concentration after 8 days.

77. **Fibonacci's Rabbits** Fibonacci posed the following problem: Suppose that rabbits live forever and that each month each pair produces a new pair that becomes productive at age 2 months. If we start with one newborn pair, how many pairs of rabbits will we have in the \( n \)th month? Show that the answer is \( F_n \), where \( F_n \) is the \( n \)th term of the Fibonacci sequence.

**Discovery • Discussion**

78. **Different Sequences That Start the Same**
(a) Show that the first four terms of the sequence \( a_n = n^2 \) are
\[
1, 4, 9, 16, \ldots
\]
(b) Show that the first four terms of the sequence \( a_n = (n-1)(n-2)(n-3)(n-4) \) are also
\[
1, 4, 9, 16, \ldots
\]
(c) Find a sequence whose first six terms are the same as those of \( a_n = n^2 \) but whose succeeding terms differ from this sequence.
(d) Find two different sequences that begin
\[
2, 4, 8, 16, \ldots
\]

79. **A Recursively Defined Sequence** Find the first 40 terms of the sequence defined by
\[
a_{n+1} = \begin{cases} 
\frac{a_n}{2} & \text{if } a_n \text{ is an even number} \\
3a_n + 1 & \text{if } a_n \text{ is an odd number}
\end{cases}
\]
and \( a_1 = 11 \). Do the same if \( a_1 = 25 \). Make a conjecture about this type of sequence. Try several other values for \( a_1 \) to test your conjecture.

80. **A Different Type of Recursion** Find the first 10 terms of the sequence defined by
\[
a_n = a_{n-1} + a_{n-2}
\]
with \( a_1 = 1 \) and \( a_2 = 1 \). How is this recursive sequence different from the others in this section?