Consider an object of mass \( m \) sliding without friction on a mountain range given by the graph of the function \( y = f(x) \), where \( x = x(t) \) is the horizontal coordinate of the object, and \( t \) is the time variable.

1. Find a second-order differential equation for \( x(t) \) that describes the motion of the object. Hint: Look at the gravitational force \( F_g \) acting on the object. Decompose it into a component \( F_p \) that’s perpendicular to the mountain range and a component \( F_T \) that’s tangent to the mountain. Only one of those two components will have an effect on the object. Decompose this one further, into a horizontal component \( F_H \) and a vertical component \( F_V \). Which of those two components will affect \( x(t) \)? Use Newton’s second axiom to write down a differential equation for \( x(t) \).

2. Recall that for small angles \( \alpha \), we have \( \sin \alpha \approx \alpha \approx \tan \alpha \) and \( \cos \alpha \approx 1 \). Assuming that \( f'(x) \) is small, use these approximations to simplify the differential equation for \( x(t) \).

3. Let \( f(x) = x^2 \). Find the general solution of the simplified equation for this choice of \( f(x) \), then find the solution that satisfies \( x(0) = 0 \) and \( x'(0) = 1 \). What kind of behavior do you expect? Does the solution agree with your expectations?