Math 415.

Topics covered by the Final exam

Section 1.1, 1.2.
• Vectors and linear combination
• Dot product

Section 1.3
• Matrices, matrix of coefficients
• Inverse matrix

Section 2.1
• Vector and matrix versions of systems of linear equations

Section 2.2, 2.3
• Elimination and elimination matrices

Section 2.4, 2.5
• Matrix operations
• Block multiplication
• Inverses

Section 2.6
• LU factorization

Section 2.7
• Transposes and permutation

Section 3.1
• Vector spaces and subspaces
• $\text{Col}(A)$

Section 3.2
• Null space
• Theorem: if $n>m$ then there must be free variables

Section 3.3
• Rank and dimension of $\text{Col}(A)$, $\text{N}(A)$

Section 3.4.
• The complete solution to $Ax=b$ (particular solution + $\text{N}(A)$)

Section 3.5
• Linear independence
• Basis
• Dimension
• Bases for $\text{Row}(A)$ and $\text{Col}(A)$

Section 3.6
• Dimension of the 4 subspaces
• Theorem: \( \text{dim}(\text{Col}(A)) + \text{dim}(\text{N}(A)) = n \) if \( A \) is an \( mxn \) matrix

Section 8.2
• Graphs and networks
• Kirchhoff’s laws and their matrix interpretations

Section 4.1
• Orthogonal subspaces
• Orthogonal complements
• Theorem: \( \text{N}(A) \) is the orthogonal complement to \( \text{Col}(A) \)
• Theorem:
  o Any \( n \) vectors that span \( \mathbb{R}^n \) must be linearly independent and hence form a basis
  o Any \( n \) linearly independent vectors in \( \mathbb{R}^n \) must span \( \mathbb{R}^n \) and hence form a basis

Section 4.2
• Projection onto a line in \( \mathbb{R}^n \)
• Projection onto a subspace of \( \mathbb{R}^n \)

Section 4.3
• Least squares approximation (the basic idea)

Section 4.4
• Orthogonal bases and the Gram-Schmidt procedure

Section 5.1
• Defining properties of determinants:
  o \( \text{Det}(I) = 1 \)
  o Row switches change the sign
  o Linearity in each row or column

Section 5.2
• Permutations and the ‘big formula’ for \( \text{det}(A) \)
• Cofactors and the cofactor expansion of \( \text{det}(A) \) along any row or column

Section 5.3
• Cramer’s rule (for solutions to \( Ax = b \))
• Formula for inverses using Cramer’s rule
• Area and volume formulae in terms of determinants

Section 6.1
• Eigenvectors and eigenvalues for \( nxn \) matrices
• The need for complex numbers

Section 6.2
• Diagonalizing a square matrix
• Examples of non-diagonalizable matrices

Section 6.3
• Application of eigenvalues/eigenvectors to systems of ordinary differential equations.

Section 6.4
• Symmetric matrices:
• Eigenvalues are real
• Can always find an orthonormal basis of eigenvectors
• Can always factor as $A=Q\Lambda Q^T$

Section 6.5
• Positive definite matrices (just the definition)

Section 6.6
• Similar matrices
• Jordan matrices and Jordan forms

Section 6.7
• Singular value decomposition (SVD) of an mxn matrix, i.e. $A=U\Sigma V^T$ where
  o $U$ is mxm, orthogonal, and its columns give bases for $\text{Col}(A)$ and $\text{Nul}(A^T)$,
  o $V$ is nxn, orthogonal, and its columns give bases for $\text{Row}(A)$ and $\text{Nul}(A)$,
  o $\Sigma$ is mxn with zeros everywhere except an rxr diagonal block in its upper left corner, where $r=\text{Rank}(A)$.

Section 7.1
• Linear transformations

Section 7.2
• Choice of bases and the matrix of a linear transformation