Math 225, Fall 2010: Midterm 2 Summary

This is a list of the main ideas and results in each section of the book.

Chapter 2: Matrix Algebra
2.1 Matrix Operations
• addition, scalar multiplication
• matrix multiplication
• Properties (see Theorem 2)
• Transpose
2.2 Inverses
• Properties of inverses (Theorem 6)
• Elementary matrices
• Algorithm for computing inverses
2.3 Characterization of invertibility in terms of:
• reduced echelon form, number of pivots, null space, linear independence of columns, column space

Chapter 3: Determinants
3.1 Introduction
• Cofactor expansion
• Determinant of a triangular matrix
3.2 Properties of det(A)
• Behavior under row operations
• Relation to invertibility
• Behavior under taking transpose
• Multiplicative properties
3.3 Cramer’s Rule, volume, and linear transformations
• Formula for solutions to Ax=b
• Formula for inverses
• Area and volume formulae
• Determinant as distortion factor for linear transformation

Chapter 4: Vector spaces
4.1 Vector spaces and subspaces
• Definitions
• Span{v1,v2, ...,vn}
4.2 Nul(A) and Col(A)
• Definitions and proof that they’re subspaces
• Kernel and range for a linear transformation
4.3 Linear independence and bases
• Definitions of generating set, linear independence, basis
• Theorem: If the vectors in a spanning set are not linearly independent then one of them can be removed.

• Bases for Nul(A) and Col(A)

4.5 Dimension

• Theorem: If a basis for a vector space has $n$ elements then and set of $k > n$ non-zero vectors must be linearly dependent.

• Theorem: Any two bases for a given vector space have the same number of vectors.

• Definition of dimension

• Theorem: If $\dim(V) = n$ then any set of $n$ non-zero vectors which generates $V$ is a basis, and any set of $n$ linearly independent vectors is a basis.

• Theorem: The dimension of a subspace is no bigger than the dimension of the whole vector space.

4.6 Rank

• Definition

• The Rank Theorem: $\text{rank}(A) + \dim \text{Nul}(A) = n$

• $\text{Rank}(A) = \dim \text{Row}(A) = \dim \text{Col}(A)$

• Theorem: $n \times n$ matrix $A$ is invertible iff $\text{rank}(A) = n$