Orthogonal projections and planes in $\mathbb{R}^3$.

1. Let $u = (2, 4), v = (3, 1),$ and $w = (4, 4)$.
   
   (a) Find the projections $\text{proj}_v(u)$ and $\text{proj}_v(w)$ of the vectors $u$ and $w$ onto $v$.
   
   (b) The orthogonal complement of $u$ with respect to $v$ is the vector
   
   \[ \text{orth}_v(u) = u - \text{proj}_v(u). \]
   
   Find $\text{orth}_v(u)$ and $\text{orth}_v(w)$. Represent all seven vectors in a figure (the three origi- 
   
   (c) Check that the complements $\text{orth}_v(u)$ and $\text{orth}_v(w)$ are both orthogonal to $v$.

2. Let $P = (1, 2, 3), Q = (0, 1, 2),$ and $u = (1, 2, 1)$.
   
   (a) Find the distance from $P$ to the line $t \mathbf{u}$. Hint: What is the closest point to $P$ on this 
   
   (b) Find the distance from $P$ to the line $Q + t \mathbf{u}$. Hint: The distance from $P$ to $Q + t \mathbf{u}$ is 
   
   the same as the distance from which point to the line $t \mathbf{u}$?

3. Let $u$ and $v$ be vectors in $\mathbb{R}^2$. Set
   
   \[ w = \text{orth}_v(u) = u - \text{proj}_v(u). \]
   
   Find $\text{proj}_w u$ in terms of $u, v,$ and $w$.

4. Let $P = (-1, 3, 0)$ and let $\mathcal{P}$ be the plane described by the equation
   
   \[-2x + y + z = 3.\]
   
   (a) Find a normal vector $n$ to the plane $\mathcal{P}$. Hint: First find a normal vector to the plane 
   through the origin $-2x + y + z = 0$. How is this normal vector related to $n$?
   
   (b) Find a point $Q$ on the plane $\mathcal{P}$. Check that if $R$ is also a point on $\mathcal{P}$, then $R - Q$ lies 
   
   on the plane through the origin
   
   \[-2x + y + z = 0.\]
   
   In other words, this says that the translate of the plane $\mathcal{P}$ by $Q$ is the plane through 
   
   (c) Find the projection $\text{proj}_n(P - Q)$.
   
   (d) Use the information from the previous parts to find the distance from $P$ to $\mathcal{P}$. 