1. (a) \[
\frac{d}{dt} (\tan t) = \frac{d}{dt} (\sin (\cos (\tan t)))
\]
\[= \cos (\cos (\tan t)) \frac{d}{dt} (\cos (\tan t))
\]
\[= \cos (\cos (\tan t)) (-\sin (\tan t)) \frac{d}{dt} (\tan t)
\]
\[= \cos (\cos (\tan t)) (-\sin (\tan t)) \sec^2 t
\]

1. (b) From \(\frac{ds}{dt} = \frac{ds}{dx} \cdot \frac{dx}{dt}\), we get
\[\frac{ds}{dt} = \frac{1}{4 \times \frac{3}{4}} \cdot \frac{f'(t)}{f(t)}
\]
But we need to make sure that \(\frac{ds}{dt}\) is a single variable function of \(t\),
So \(\frac{ds}{dt} = \frac{1}{4 \left[ \ln (f(t)) \right]^{3/4}} \cdot \frac{f'(t)}{f(t)}
\]

2. (a) Note that \((\frac{x}{5})^2 + (\frac{y}{3})^2 = \sin^2 (3t) + \cos^2 (3t) = 1\). So this parametrizes
(at least part of) the ellipse \((\frac{x}{5})^2 + (\frac{y}{3})^2 = 1\).
By examining differing values of \(t\) in \(0 < t < \frac{2\pi}{3}\), we see that this
parametrization travels the ellipse in a clockwise fashion exactly once.
\[
t = 0 : (x(0), y(0)) = (0, 3)
\]
\[
t = \frac{\pi}{6} : (x(\frac{\pi}{6}), y(\frac{\pi}{6})) = (5, 0)
\]
\[
t = \frac{\pi}{3} : (x(\frac{\pi}{3}), y(\frac{\pi}{3})) = (0, -3)
\]
\[
t = \frac{\pi}{2} : (x(\frac{\pi}{2}), y(\frac{\pi}{2})) = (-5, 0)
\]
If we let \( t \) vary between 0 and \( 2\pi \), we will traverse the ellipse 3 times.

2. (b) \[ \text{Arc length} \quad s = \int_{a}^{b} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} \, dt \]
\[ = \int_{0}^{2\pi/3} \sqrt{(15 \cos(3t))^2 + (-9 \sin(3t))^2} \, dt \]

2. (c)

If we let \( x = 4 \cos t \) and \( y = 4 \sin t \), then \( x^2 + y^2 = (4 \cos t)^2 + (4 \sin t)^2 = 16. \)

Moreover, as \( t \) increases, this parametrization traverses the circle in a counterclockwise fashion:
\( t = 0 : \quad (x(0), y(0)) = (4, 0) \)
\( t = \frac{\pi}{2} : \quad (x(\frac{\pi}{2}), y(\frac{\pi}{2})) = (0, 4) \)
\( t = \pi : \quad (x(\pi), y(\pi)) = (-4, 0) \)
\( t = \frac{3\pi}{2} : \quad (x(\frac{3\pi}{2}), y(\frac{3\pi}{2})) = (0, -4) \)
\( t = 2\pi : \quad (x(2\pi), y(2\pi)) = (4, 0) \).

To ensure that we travel the curve only once, we restrict \( t \) to the interval \([0, 2\pi]\).

So the parametrization is \( \begin{align*}
  x &= 4 \cos t \\
  y &= 4 \sin t
\end{align*} \), when \( 0 \leq t < 2\pi \).

3. (a) First, we find the critical points of \( f(x) \).
\[ f'(x) = 4x^3 - 16x \]
\[ f'(x) = 0 \quad \text{when} \quad 4x^3 - 16x = 0 \]
\[ 4x(x^2 - 4) = 0 \]
\[ 4x(x - 2)(x + 2) = 0. \]
Hence $f'(x) = 0$ when $x = 0$, $x = 2$, or $x = -2$.

Now apply the 2nd Derivative Test to the three critical points:

From $f''(x) = 12x^2 - 16$, we get

$f''(0) = -16 < 0$, so $y = f(x)$ is concave down at the point $(0, f(0))$.

So a local max occurs at $(0, 10)$.

$f''(-2) = 32 > 0$, so $y = f(x)$ is concave up at the point $(-2, f(-2))$.

A local min occurs at $(-2, -6)$.

$f''(2) = 32 > 0$, so $y = f(x)$ is concave up at the point $(2, f(2))$.

A local min occurs at $(2, -6)$.

3. (b) First, find the critical points of $h(s)$.

$h'(s) = 4s^3 + 12s^2$.

Then $h'(s) = 0$ when $4s^3 + 12s^2 = 0$.

$4s^2(s + 3) = 0$.

So $h'(s) = 0$ when $s = 0$ and $s = -3$.

For the 1st Derivative Test, we need to determine if $h$ is increasing or decreasing on the intervals $(-\infty, -3)$, $(-3, 0)$, and $(0, \infty)$.

On $(-\infty, -3)$ choose any test point (for example, choose $s = -1000$). The sign of $h'(s) = 4s^3 + 12s^2 < 0$ on this interval. Hence $h(s)$ is decreasing on $(-\infty, -3)$.

On $(-3, 0)$ choose any test point (e.g., choose $s = -1$). The sign of $h'(s) = 4s^3 + 12s^2 > 0$ on this interval. Hence $h(s)$ is increasing on $(-3, 0)$.

On $(0, \infty)$ choose any test point (e.g., choose $s = 1000$). The sign of $h'(s) = 4s^3 + 12s^2 > 0$ on this interval. Hence $h(s)$ is increasing on $(0, \infty)$.
Since at \( s = -3 \) the function changes from decreasing to increasing, the function must have obtained a local min at \( s = -3 \).

At \( s = 0 \), neither a max or a min occurs in the value of \( h \).

3. (c) When \( s = -3 \), \( h''(-3) = 36 > 0 \). A local min occurs when \( s = -3 \) by the 2nd Derivative Test.

When \( s = 0 \), \( h''(0) = 0 \). The 2nd Derivative Test is inconclusive. The graph of \( y = h(s) \) has no concavity at \((0, h(0))\). Without more information (the 1st Derivative Test), we are unable to identify \((0, h(0))\) as a local max, min, or a point of inflection.

4. (a) Recall that in Calc I and II, the "best linear approximation" is synonymous with the equation of the tangent line or the 1st-order Taylor polynomial.

Here, \( f'(x) = 2xe^{-x} + x^2(-e^{-x}) \).

Since \( f'(0) = 0 \), the tangent line has no slope at \((0, f(0)) = (0, 0)\).

The equation of the tangent line is \( y = 0 \).

4. (b) By definition, the second-order Taylor polynomial at \( x = 0 \) is

\[
T_2(x) = f(0) + \frac{f'(0)(x-0)}{1!} + \frac{f''(0)(x-0)^2}{2!}
\]

Since \( f''(x) = 2e^{-x} - 4xe^{-x} + x^2e^{-x} \), we compute that \( f''(0) = 2 \).

Hence \( T_2(x) = 0 + \frac{2(x-0)}{1!} + \frac{2(x-0)^2}{2!} = x^2 \).
4. (c) The second-order Taylor polynomial is the best quadratic approximation to the curve \( y = f(x) \) at the point \((0, f(0))\). Since \( T_2(x) = x^2 \) clearly has a local minimum at \((0,0)\), and \((0,0)\) is the location of a critical point of \( f \), then \( f \) must also have a local minimum at \((0,0)\).

5. (a)

5. (b) Let \( u = x^2 \). Then \( du = 2x \, dx \), so the integral becomes

\[
\int_{0}^{\sqrt{3\pi}} 2x \cos(x^2) \, dx = \int_{0}^{3\pi} \cos u \, du.
\]

5. (c) \[
\int_{0}^{\sqrt{3\pi}} 2x \cos(x^2) \, dx = \int_{0}^{3\pi} \cos u \, du
\]

\[
= \left[ \sin u \right]_{u = 0}^{u = 3\pi}
\]

\[
= \sin 3\pi - \sin 0
\]

\[
= 0.
\]