You may use a single (one-sided) page of hand-written notes prepared by you and approved by the instructor in advance.
**Problem 1.** (20 pts.) Evaluate the following expressions:

(a) (4 pts.) $99 \mod 100 = \underline{}$

(b) (4 pts.) $100 \mod 99 = \underline{}$

(c) (4 pts.) $-99 \mod 100 = \underline{}$

(d) (4 pts.) $-100 \mod 99 = \underline{}$

(e) (4 pts.) $99 \mod 99 = \underline{}$
Problem 2. (20 pts.) The International Fixed Calendar (the IFC, for short) is a proposal for calendar reform designed by M.B. Cotsworth in 1902.

The IFC divides the year into 13 months of 28 days each; every month starts on Sunday. Twelve of the months retain their names and order (January, February, March, etc.), with an extra month, called Sol, inserted between June and July. As the 13 months amount to a total of $28 \cdot 13 = 364$ days, an extra day, called Year Day, is added as a holiday at the end of each year.

According to the IFC, today is November 27.

(a) (5 pts.) What day of the week is today, according to the IFC?

(b) (5 pts.) What date and day of the week will it be in 10 days, according to the IFC?

(c) (5 pts.) What date and day of the week will it be in 100 days, according to the IFC?

(d) (5 pts.) How many days ago was June 1, according to the IFC?
Problem 3. (20 pts.)

(a) (10 pts.) Convert the following numbers to the decimal system:

(a.1) (5 pts.) $123_4 = \underline{________} \\

(a.2) (5 pts.) $AA_{16} = \underline{________} \\

(b) (10 pts.) Convert the following numbers to the binary system:

(b.1) (5 pts.) $11 = \underline{________} \\

(b.2) (5 pts.) $101 = \underline{________} \\


Problem 4. (20 pts.) You are planning to use the RSA with $N = 85$.

For the given pairs of numbers, determine if they form a valid public/private key pair.

(a) (10 pts.) $e = 5$, $d = 13$.

(b) (10 pts.) $e = 11$, $d = 12$. 
Problem 5. (20 pts.) Five friends are planning a trip to Europe and holding an election in order to decide which city to visit first. Four of them have already cast their votes as follows:

<table>
<thead>
<tr>
<th></th>
<th>Fred</th>
<th>Daphne</th>
<th>Shaggy</th>
<th>Velma</th>
<th>Scooby</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top choice</td>
<td>Amsterdam</td>
<td>Rome</td>
<td>London</td>
<td>London</td>
<td></td>
</tr>
<tr>
<td>Second choice</td>
<td>Munich</td>
<td>Munich</td>
<td>Amsterdam</td>
<td>Paris</td>
<td></td>
</tr>
<tr>
<td>Third choice</td>
<td>Paris</td>
<td>London</td>
<td>Munich</td>
<td>Munich</td>
<td></td>
</tr>
<tr>
<td>Fourth choice</td>
<td>London</td>
<td>Amsterdam</td>
<td>Paris</td>
<td>Rome</td>
<td></td>
</tr>
<tr>
<td>Fifth choice</td>
<td>Rome</td>
<td>Paris</td>
<td>Rome</td>
<td>Amsterdam</td>
<td></td>
</tr>
</tbody>
</table>

The winner is chosen using the Borda method.

(a) (4 pts.) Which option wins if Scooby’s vote is not counted?

(b) (8 pts.) Here is how Scooby ranks the options, from most to least preferred:


Which option will win if Scooby votes honestly, i.e., gives his real list of preferences?

(c) (8 pts.) Can Scooby vote in such a way as to make his top choice, Amsterdam, the winner?
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