MATH 181: EXAM 2

Name: ____________________________________________

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You may use a single (one-sided) page of hand-written notes prepared by you and approved by the instructor in advance.

Please **do not** use a calculator!
Problem 1. (20 pts.) A diamond ring is bought for $500, and its value increases by 10% each year.

(a) (5 pts.) How much will the ring cost a year from now?

(b) (5 pts.) How much will the ring cost two years from now?

(c) (5 pts.) How much will the ring cost \( n \) years from now?

(d) (5 pts.) If the ring’s value increased by \( p \)% each year, then how much would it cost in 10 years?
Problem 2. (10 pts.) Evaluate the following expressions:

(a) (2 pts.) \( \log_2(16) = \)

(b) (2 pts.) \( \log_5(5) = \)

(c) (2 pts.) \( \log_{10}(1000) = \)

(d) (2 pts.) \( \log_{25}(5) = \)

(e) (2 pts.) \( \log_8(16) = \)
Problem 3. (10 pts.) The diagrams shown on page 5 illustrate the growth of the world population. Explain why these diagrams look so different even though they represent the same data.
Problem 4. (20 pts.)

(a) (10 pts.) Find the continued fraction expansion of the number $25/11$:

$$\frac{25}{11} =$$

(b) (10 pts.) Find the convergents of the continued fraction that you obtained in part (a).
Problem 5. (20 pts.)

(a) (10 pts.) In a substitution cipher, each letter in the top row is replaced by the corresponding letter in the bottom row:

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
MATHZYXWUSRQPONLKJIGFEDCB
```

(a.1) (5 pts.) Decrypt the following message:

```
AZMQQ ZGN
```

(a.2) (5 pts.) Encrypt the following message:

```
ILLIN OIS
```

(b) (5 pts.) Write down the substitution alphabet for the Caesar cipher with shift 5.

```
ABCDEFGHIJKLMNOPQRSTUVWXYZ
```

(c) (5 pts.) Use the Caesar cipher with shift 5 to encode the following message:

```
MEETA TDAWN
```
Problem 6. (20 pts.) One of the bits in the message 0000110, encoded using the Hamming code, may be erroneous. Correct the error if there is one and extract the original four-bit message.
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