You may use a single (one-sided) page of hand-written notes prepared by you and approved by the instructor in advance.
Problem 1. (20 pts.) Answer the questions about the graph below (no justification is needed).

(a) (2 pts.) How many vertices does this graph have? _____
(b) (2 pts.) How many edges does it have? _____
(c) (2 pts.) Is it connected? _____
(d) (2 pts.) How many components does it have? _____
(e) (2 pts.) What is the degree (valence) of vertex A? _____
(f) (2 pts.) What is the degree of vertex D? _____
(g) (2 pts.) How many vertices in this graph have even degrees? _____
(h) (2 pts.) Is $B – C – D – E – B$ a cycle? _____
(j) (2 pts.) Write down a path from $F$ to $G$ using the fewest edges: ____________________
Problem 2. (20 pts.) Are the graphs in each of the following pairs isomorphic? If yes, number their vertices to show an isomorphism. If no, explain what is different between them.

(a) (10 pts.)

\begin{itemize}
  \item[(a.1)]
  \begin{itemize}
    \item \[\text{Graph 1}\]
  \end{itemize}
  \begin{itemize}
    \item \[\text{Graph 2}\]
  \end{itemize}
\end{itemize}

(b) (10 pts.)

\begin{itemize}
  \item[(b.1)]
  \begin{itemize}
    \item \[\text{Graph 3}\]
  \end{itemize}
  \begin{itemize}
    \item \[\text{Graph 4}\]
  \end{itemize}
\end{itemize}
Problem 3. (20 pts.)

(a) (5 pts.) Draw a graph with 5 vertices, 4 edges, and 2 components.

(b) (5 pts.) Draw a graph with 6 vertices in which every vertex has degree 2.

(c) (5 pts.) Draw a graph that contains an Euler circuit but no Hamiltonian cycle.

(d) (5 pts.) Draw a graph that contains a Hamiltonian cycle but no Euler circuit.
Problem 4. (20 pts.)

(a) (5 pts.) Describe how Kruskal’s algorithm works.

(b) (10 pts.) Use Kruskal’s algorithm to find a minimum weight spanning tree in the given weighted graph. Show all the intermediate steps.

(c) (5 pts.) Construct a TSP route using the spanning tree that you found in part (a). What is the weight of the resulting route?
Problem 5. (10 pts.) The table below shows the mileage between four cities in Illinois:

<table>
<thead>
<tr>
<th></th>
<th>Champaign</th>
<th>Eldorado</th>
<th>Geneva</th>
<th>Paris</th>
</tr>
</thead>
<tbody>
<tr>
<td>Champaign</td>
<td>—</td>
<td>177</td>
<td>160</td>
<td>67</td>
</tr>
<tr>
<td>Eldorado</td>
<td>177</td>
<td>—</td>
<td>352</td>
<td>144</td>
</tr>
<tr>
<td>Geneva</td>
<td>160</td>
<td>352</td>
<td>—</td>
<td>198</td>
</tr>
<tr>
<td>Paris</td>
<td>67</td>
<td>144</td>
<td>198</td>
<td>—</td>
</tr>
</tbody>
</table>

Represent this information by drawing a weighted graph on four vertices.
Problem 6. (10 pts.) Consider the graph below.

(a) (2 pts.) What is the chromatic number of this graph? $$\text{_____}$$

(b) (2 pts.) In the space below, redraw this graph so that no two edges cross.

(c) (2 pts.) How many faces does your drawing in part (b) have? $$\text{_____}$$

(d) (2 pts.) In the space below, draw the dual graph corresponding to your drawing in part (b).

(e) (2 pts.) How many faces does the dual graph that you drew in part (d) have? $$\text{_____}$$
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