Theorem 1 (Arrow’s Theorem). Suppose that a number of candidates are running in an election. Each voter ranks all the candidates from the most preferred to the least preferred. A voting system is supposed to output, based on these individual rankings, an “aggregate” ranking of the candidates. Call a voting system “reasonable” if it satisfies the following three requirements:

Non-dictatorship: There is no single voter whose ballot determines the outcome of the election.

Unanimity*: For any two candidates A and B, if every voter ranks A higher than B, then A is ranked higher than B in the outcome of the election.

Independence of irrelevant alternatives: For any two candidates A and B, the position of A relative to B in the final ranking is determined solely by the position of A relative to B in the voter’s rankings.

If the number of candidates is at least 3, then there is no “reasonable” voting system.

*Also called weak Pareto efficiency.

The first two requirements in Arrow’s theorem are very natural. On the other hand, independence of irrelevant alternatives seems like an unnecessarily strong restriction. The motivation behind it is roughly as follows. Imagine we are electing a president. We may assume that there is some objective measure of how well a given candidate is qualified for that role. Our goal then is to find which of the candidates is the most qualified. For simplicity, suppose that the qualifications of a candidate can be measured by a number from 0 to 100. So if candidate A scores, say, 90, while candidate B scores 80, then A is better than B and should be ranked higher by our voting system. Note that the relative ranking of A and B only depends on their own qualifications; in other words, any other candidates are irrelevant to the decision between A and B.

This logic ignores the fact that the only information we have available are the voter’s ballots; indeed, the positions of A and B relative to the other candidates may help us ascertain their personal qualifications. For instance, suppose that one ballot ranks A and B as the first and the second choice respectively, while another ballot ranks A first and B last. Both of them indicate a preference for A over B; but the latter one also shows that the gap between A’s and B’s qualifications is much higher.

Nevertheless, there exist several other results in the spirit of Arrow’s theorem that assert the impossibility of creating a “reasonable” voting system under some less contentious assumptions. The following is an example:

Theorem 2 (The Duggan–Schwartz theorem). Suppose that a number of candidates are running in an election. Each voter ranks all the candidates from the most preferred to the least preferred. A voting system is supposed to output, based on these rankings, a set of winning candidates (i.e., more than one candidate may be declared a winner). Call such a system “reasonable” if it satisfies the following three requirements:

No weak dictator: There is no single voter whose most preferred candidate is always among the winners.

Every candidate is viable: Any candidate has a chance of being the sole winner.

Non-manipulability: For every voter, it is always best to honestly represent his or her preferences on the ballot.

If the number of candidates is at least 3, then there is no “reasonable” voting system.