Checksums

**Vocabulary:** check digits.

**Luhn’s formula** is a method for computing a check digit that is used to validate a variety of identification numbers, such as credit card numbers, National Provider Identifier numbers, Canadian Social Insurance numbers, etc. Luhn’s method detects an error in any single digit or a transposition of two adjacent digits (apart from $09 \leftrightarrow 90$).

Here is how Luhn’s formula works: The check digit is the last digit of the number. To validate the number, first double every other digit starting from the second right-most one. Then add all the resulting digits together. The final outcome must be divisible by 10 (i.e., its last digit must be 0).

**Example:**

\[
\begin{array}{cccccccccccccccc}
6 & 0 & 1 & 1 & 1 & 5 & 4 & 8 & 8 & 6 & 2 & 2 & 3 & 0 & 0 & 5 \\
12 & 0 & 2 & 1 & 2 & 5 & 8 & 8 & 16 & 6 & 4 & 2 & 6 & 0 & 0 & 5 \\
\end{array}
\]

The sum of the digits in the second line is $1+2+0+2+1+2+5+8+8+1+6+6+4+2+6+0+0+5 = 59$. Since it is not divisible by 10, the credit card number was invalid.