**The Four Color Theorem**

$k$-Col: “Is a given graph $k$-colorable?”

It is easy to determine if a given graph is 1-colorable or 2-colorable. Formally, the decision problems $1$-Col and $2$-Col belong to the class $P$.

**Theorem** (Karp 1972). *For each $k \geq 3$, the decision problem $k$-Col is NP-complete.*

By a **plane map** we mean a partition of the plane into finitely many contiguous regions. Two regions are **adjacent** if they share a stretch of their borders (just touching is not enough!).

**Theorem** (Appel–Haken 1976). *The regions of any plane map can be colored with no more than four colors so that no two adjacent regions have the same color.*

**Historical note:** The **Four Color Problem** was suggested by Francis Guthrie in 1852, when he was trying to color the map of counties of England. In 1879, Alfred Kempe published an alleged proof that was widely accepted. Kempe’s “proof” was shown wrong by Percy Heawood in 1890, who modified Kempe’s arguments to prove that **five** colors suffice for every plane map. The problem was finally settled by Kenneth Appel and Wolfgang Haken in 1976 at UIUC. Their proof is notable for being the first proof of a major result in mathematics that required the use of a **computer** to facilitate a massive amount of computations. To this day, no human-checkable proof of the Four Color Theorem is known.