1. \[ C_0 = 2, \quad C_1 = \frac{P_1}{q_1} = \frac{9a^2 + 1}{a} = \frac{2 \cdot 1 + 1}{1} = 3 \]

\[ C_2 = \frac{P_2}{q_2} = \frac{a_2 P_1 + P_0}{a_2 q_1 + q_0} = \frac{4 \cdot 3 + 2}{4 \cdot 1 + 1} = \frac{14}{5} \]

\[ C_3 = \frac{P_3}{q_3} = \frac{a_3 P_2 + P_1}{a_3 q_2 + q_1} = \frac{3 \cdot 14 + 3}{3 \cdot 5 + 1} = \frac{45}{16} \]

\[ C_4 = \frac{P_4}{q_4} = \frac{a_4 P_3 + P_2}{a_4 q_3 + q_2} = \frac{3 \cdot 45 + 14}{3 \cdot 16 + 5} = \frac{149}{53} = [2, 4, 3, 3] \]

2. a. \[ 335 = 3^3 \cdot 5, \quad 3 \equiv 3 (\text{mod} 4), \text{its power is odd} \ldots \therefore \text{not representable} \]

b. \[ 181 \equiv 1 (\text{mod} 4) \text{ is prime} \ldots \therefore \text{representable} \]

c. \[ 6 \text{ and } 8 \text{ are even powers of primes} \equiv 3 (\text{mod} 4) \ldots \therefore \text{representable} \]

3. If there are integer solutions, then \(-2 \equiv (2y)^2 \text{ (mod 5)}\) has solutions. Thus, \((-\frac{2}{5}) = 1 \). But

\[ (-\frac{2}{5}) = (-\frac{1}{5}) \left( \frac{2}{5} \right) = 1 \cdot -1 = -1 \]

as \( 5 \equiv 1 \text{ (mod 4)} \) and \( 5 \equiv 5 \text{ (mod 8)} \). \( \therefore \text{no solutions} \)

4. Let \( (x, y, z) \) be a primitive Pythagorean triple, i.e. \( x^2 + y^2 = z^2 \), \( (x, y, z) = 1 \). Note \( z \equiv 0 \text{ (mod 3)} \), because \( x^2 + y^2 \equiv 0 \text{ in all cases} \). Thus, \( z^2 \equiv 1 \text{ (mod 3)} \). If \( x \equiv \pm 1 \text{ (mod 3)} \), \( y \equiv \pm 1 \text{ (mod 3)} \), then \( x^2 + y^2 \equiv 2 \text{ (mod 3)} \neq 1 \text{ (mod 3)} \). Thus, exactly one of \( (x, y) \) is a multiple of 3 and the other is \( \equiv \pm 1 \text{ (mod 3)} \).

5. Suppose \( z \) is not primitive. Then \( (z')^a \equiv 1 \text{ (mod p)} \) for some \( a < p-1, \ a \text{ (p-1)} \). Now

\[ n z' \equiv 1 \text{ (mod p)} \]

\[ (n z')^a \equiv 1 \text{ (mod p)} \]

\[ \Rightarrow n^a \equiv 1 \text{ (mod p)}, \text{ since } (z')^a \equiv 1 \text{ (mod p)} \]

\[ \rightarrow n^a \equiv 1 \text{ (mod p)}, \text{ since } n^a \equiv 1 \text{ (mod p)} \]

as the least power, i.e., \( p-1 \) so that \( n^{p-1} \equiv 1 \text{ (mod p)} \)
6. \( a_0 = \lceil \sqrt{5} \rceil = 2 \)
\[
d_1 = \frac{4}{a_0 - a_0} = \frac{4}{\sqrt{5} - 2} = \frac{4}{\sqrt{5} - 2}\]
\[
a_1 = \lfloor \alpha_1 \rfloor = \lfloor \sqrt{5} + 2 \rfloor = 4\]
\[
d_2 = \frac{4}{a_1 - a_1} = \frac{4}{\sqrt{5} + 2 - 4} = \frac{4}{\sqrt{5} - 2}\]
\[
a_2 = \lfloor \alpha_2 \rfloor = \lfloor \sqrt{5} + 2 \rfloor = 4\]

Assume that \( \alpha_n = \sqrt{5} + 2, a_n = 4 \). Then
\[
a_{n+1} = \frac{4}{a_n - a_n} = \frac{4}{\sqrt{5} + 2 - 4} = \frac{4}{\sqrt{5} - 2}\]
\[
\therefore \sqrt{5} = \lfloor 2, 4, 4, 4, \ldots \rfloor.
\]

7. As \( n \) runs through a complete residue system modulo \( p \), \( x^{-1} \) does as well. Thus, replace \( x \) by \( x^{-1} \) in the definition of \( G(\beta, x) \).
\[
G(\beta, x) = \sum_{x \equiv n \mod p} x(a^{-1} x) e^{2 \pi i x^{-1} \beta / p} = \chi(a^{-1}) \sum_{x \equiv n \mod p} x(a^{-1}) e^{2 \pi i x^{-1} \beta / p} = \chi(a^{-1}) G(\beta, x) \]
\[
\quad \quad \text{as } \chi \text{ is multiplicative},
\]
\[
\text{because } \chi(a^{-1}) \chi(a) = 1 = \overline{\chi(a)} \chi(a) = 1.
\]

8. Since \( p \equiv 1 \pmod{4} \), \( -1 \) is a quadratic residue, and so
\[
\exists x, 0 \leq x \leq \frac{p-1}{2}, \text{ such that } x^2 \equiv -1 \pmod{p}
\]
\[
x^2 + 1 = k p, \ k \in \mathbb{Z}^+
\]
\[
k p < \left( \frac{p}{2} \right)^2 + 1 < p \quad \Rightarrow \quad k < p
\]
\[
\therefore \quad x^2 + y^2 + 1 = k p
\]
where here we can take \( a_1 = 0 \).