2.4.42 (a) \( y_1(t) = 1 - t, \quad y_1'(t) = -1, \quad y_1(2) = 1 - 2 = -1 \)
\[-t + \frac{t^2 + 4(1 - t)}{2} = -t + \frac{(t - 2)^2}{2} = \frac{-t + t - 2}{2} = -1\]
Thus, \( y_1(t) \) is a solution of
\[y' = -\frac{t + \sqrt{t^2 + 4y}}{2}, \quad y(2) = -1. \quad (\star)\]
\[y_2(t) = -t^2/4, \quad y_2'(t) = -t/2, \quad y_2(2) = -2/2 = -1 \]
\[-\frac{t + \sqrt{t^2 + 4 \cdot -t^2/4}}{2} = -\frac{t + 0}{2} = -\frac{t}{2}\]
Thus, \( y_2(t) \) is also a solution of \((\star)\).

(b) Here \( f(t, y) = -\frac{t + \sqrt{t^2 + 4y}}{2} \)
\[
\frac{2f}{2y} = \frac{1}{2} \cdot \frac{4}{(t^2 + 4y)^{1/2}} = \left(\frac{t^2 + 4y}{2}\right)^{-1/2}
\]
At \((2, -1)\),
\[t^2 + 4y = 4 - 4 = 0.\]
Thus, \( \frac{2f}{2y} \) does not exist at \((2, -1)\). In particular, \( \frac{2f}{2y} \) is not continuous at \((2, -1)\), and so the hypotheses of Theorem 2.4.2 are not satisfied.

(c) Let \( y_3(t) = ct + c^2 \). So, \( y_3'(t) = c \cdot \frac{2t + 4c}{2} = -t + \sqrt{t^2 + 4(ct + c^2)} = -t + \frac{(t + 2c)^2}{2} = \frac{-t + t + 2c}{2}\)
Thus, the differential equation \((\star)\) is satisfied. If \( c = -1 \), we obtain the solution \( y_3(t) \) of part (a).
\[y_3(t) = ct + c^2 \text{ is linear in } t, \quad \text{while } y_2(t) = -t^2/4 \text{ is quadratic in } t. \]
Thus, \( c \) such that \( ct + c^2 = -t^2/4. \)
Lastly, observe that the expression under the square root must be nonnegative. Thus, we need \( t + 2c \geq 0 \), or \( t \geq -2c \).
2.8. If \( x = 0 \), \( \Phi_n(x) = 0 \) for all \( n \geq 0 \). If \( x > 0 \), then
\[
\lim_{n \to \infty} \Phi_n(x) = \lim_{n \to \infty} \frac{2nx}{e^{nx^2}} = 0
\]
by L'Hôpital's Rule. Thus,
\[
\int_0^1 \lim_{n \to \infty} \Phi_n(x) \, dx = \int_0^1 0 \, dx = 0.
\]
(b) \[
\int_0^1 2nx e^{-nx^2} \, dx = -e^{-nx^2}\bigg|_0^1 = -e^{-n} + 1
\]
Thus,
\[
\lim_{n \to \infty} \int_0^1 \Phi_n(x) \, dx = \lim_{n \to \infty} (-e^{-n} + 1) = 1,
\]
\[
\int_0^1 \lim_{n \to \infty} \Phi_n(x) \, dx = 0 \neq \lim_{n \to \infty} \int_0^1 \Phi_n(x) \, dx = 1.
\]

2.9.47
\[
y'' + (y')^2 = 2e^{-y}
\]
Let \( v = y' \) and regard \( y \) as the independent variable.
\[
y'' = v' = \frac{dv}{dy} \frac{dy}{dt} = \frac{dv}{dy} v
\]
Thus,
\[
\frac{dv}{dy} v + v^2 = 2e^{-y}
\]
\[
\frac{dv}{dy} + v = 2e^{-y}v^{-1}
\]
This is a Bernoulli equation with \( n = -1 \). Thus, let
\( u = v^{-2} \). Thus,
\[
\frac{1}{2v} \frac{du}{dy} + uv^{-1} = 2e^{-y}v^{-1}
\]
Cancel \( v^{-1} \). Multiply both sides by \( 2v \).
\[
\frac{du}{dy} + 2uv = 4e^{-y}
\]
This is a linear, first order I.C. An integrating factor is
\[
e^{-2y}.
\]
Thus, 
\[ e^{-y} \frac{du}{dy} + 2ue^{-y} = ke^{-y} \]

or
\[ \frac{d}{dy} (ue^{-y}) = ke^{-y} \]

\[ u e^{-y} = ke^{-y} + 4c_1, \] (linear constant of integration $\sim 4c_1$, on factor $4$)

\[ u = ke^{-y} + 4c_1 e^{-2y} \]

or
\[ u^2 = 4e^{-2y} + 4c_1 e^{-2y} \]
\[ \frac{du}{dt} = u = \sqrt{4e^{-2y} + 4c_1 e^{-2y}} = 2e^{-y} + c_1 e^{-2y}, \]

separately variables to get

\[ \frac{dy}{2 \sqrt{e^{-y} + c_1 e^{-2y}}} = dt \]

\[ \frac{1}{2} \int \frac{dy}{\sqrt{e^{-y} + c_1 e^{-2y}}} = t + c_2 \]

\[ \frac{1}{2} \int \frac{e^{y} dy}{\sqrt{e^{-y} + c_1}} = t + c_2 \]

Let $u = e^{y}$, so $du = e^{y} dy$. Thus,

\[ \frac{1}{2} \int \frac{e^{y} dy}{\sqrt{e^{y} + c_1}} = \frac{1}{2} \int \frac{du}{\sqrt{u + c_1}} = \frac{1}{2} \left( \frac{u + c_1}{1/2} \right) = (u + c_1)^{1/2} \]

Thus,

\[ (u + c_1)^{1/2} = t + c_2 \]

\[ u + c_1 = (t + c_2)^{2} \]

\[ e^{y} + c_1 = (t + c_2)^{2} \]

\[ e^{y} = (t + c_2)^{2} - c_1 \]

\[ = (t + c_2)^{2} + C_1 \] (\( C_1 = -c_1 \))