$y' = \dfrac{t^2}{y(1+t^3)}$, $y(0) = y_0 > 0$. (*)

$t = -1$ is a point of discontinuity. Thus, we should assume that either $t < -1$ or $t > -1$. Since our initial condition is at $t = 0$, we need to assume that $t > -1$. Also $y = 0$ is a discontinuity. Thus, we must assume either $y > 0$ or $y < 0$. Since $y_0 > 0$, we must assume that $y > 0$. We solve (*) by separating variables.

$$y\,dy = \frac{t^2 dt}{1+t^3} \Rightarrow \frac{y^2}{2} = \frac{1}{3}\log(1+t^3) + c$$

If $t = 0$, $\dfrac{y_0^2}{2} = \dfrac{1}{3}\cdot 0 + c \Rightarrow c = \dfrac{y_0}{2}$.

Thus, $$y^2 = \frac{2}{3}\log(1+t^3) + y_0^2$$

$$\Rightarrow y = \pm\sqrt{\frac{2}{3}\log(1+t^3) + y_0^2}.$$

Since $y_0 > 0$ at $t = 0$, we need to take the + sign. Thus,

$$y = \sqrt{\frac{2}{3}\log(1+t^3) + y_0^2}.$$

The function under the square root must be ≥ 0, i.e.

$$\frac{2}{3}\log(1+t^3) + y_0^2 \geq 0$$

$$\Rightarrow \log(1+t^3) \geq -\frac{3y_0^2}{2}$$

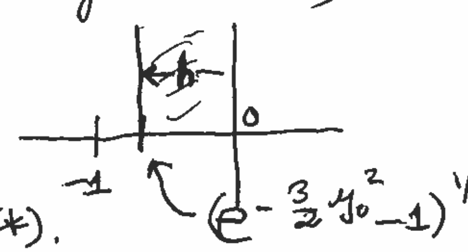
$$\Rightarrow 1+t^3 > \exp\left(-\frac{3}{2}y_0^2\right) \Rightarrow t^3 > \exp\left(-\frac{3}{2}y_0^2\right) - 1$$

We now take the cube root of both sides of (**). Note that the right side is < 0. Thus, the cube root will be negative. Thus,

$$t > \left(e^{-\frac{3}{2}y_0^2} - 1\right)^{1/3}.$$

Hence,

$$h = \left(1 - e^{-\frac{3}{2}y_0^2}\right)^{1/3}.$$



Hence, $\exists$ a unique solution to (*).
The solution is given by (±), and it is valid for $-h < t < h$, where h is given above.