\[ y' = \frac{t^2}{y^3(t+1)}, \quad y(0) = y_0 > 0. \quad (x) \]

\( t = -1 \) is a point of discontinuity. Thus, we should assume that

either \( t < -1 \) or \( t > -1 \). Since our initial condition is at \( t = 0 \), we

need to assume that \( t > -1 \). Also \( y = 0 \) is a discontinuity. Thus, we

must assume either \( y > 0 \) or \( y < 0 \). Since \( y_0 > 0 \), we must assume

that \( y > 0 \). We solve (x) by separating variables.

\[ y \cdot dy = \frac{t^2 dt}{t+1} \Rightarrow \frac{y^2}{2} = \frac{1}{3} \log(a+t^3) + c \]

\( y/t = 0 \), \( \frac{y}{y_0} = \frac{1}{3} \cdot 0 + c \Rightarrow c = \frac{y_0}{2} \).

Thus,

\[ y = \pm \sqrt{\frac{2}{3} \log(a+t^3) + y_0^2} \]

Since \( y_0 > 0 \) at \( t = 0 \), we need to take the + sign. Thus,

\[ y = \sqrt{\frac{2}{3} \log(a+t^3) + y_0^2} \]

The function under the square root must be \( > 0 \), i.e.

\[ \frac{2}{3} \log(a+t^3) + y_0^2 \geq 0 \]

\[ \Rightarrow \log(a+t^3) \geq \frac{-3y_0^2}{2} \]

\[ \Rightarrow a+t^3 > \exp\left(-\frac{3}{2}y_0^2\right) \Rightarrow t^3 > \exp\left(-\frac{3}{2}y_0^2\right) - 1 \]

We now take the cube root of both sides of (x*). Note that the right side is \( < 0 \). Thus, the cube root will be negative. Thus,

\[ t > \left(e^{-\frac{3}{2}y_0^2} - 1\right)^{\frac{1}{3}} \]

Hence,

\[ h = \left(1 - e^{-\frac{3}{2}y_0^2}\right)^{\frac{1}{3}} \]

Hence, \[ h \] is a unique solution to (x*).

The solution is given by (x), and it is valid for

\(-h < t < h\), where \( h \) is given above.