Why are wise few, fools numerous in the excesse,  
'Cause, wanting number, they are numberlesse.  


SHOW ALL WORK. INDICATE ALL REASONING.

1. Find the general solution of the differential equation  
   \[ y' - \frac{2}{t} y = t^5, \quad t > 0. \]

2. Find the general solution of  
   \[ (3x^2y^2 + x^2) + (2x^3y + y^2)y' = 0. \]
3. a. Find the general solution of

$$y' = \frac{t}{y^3(t^2 - 4)}, \quad t > 0, \quad (1)$$

and determine where your solution is valid.

b. Find the particular solution of (1) satisfying the initial condition $y(\sqrt{5}) = 2$.

c. Determine the maximum value of $h$ such that your solution is valid for

$$\sqrt{5} - h < \sqrt{5} < \sqrt{5} + h.$$ 

4. On his first exam in extraordinary differential equations, Jasper Figworthy, a student of dynamic engineering at Foghead State University in Old Hogwash Junction, claimed that the initial value problem

$$y' = \cos(y^2 e^t), \quad y(0) = \pi^{13}, \quad (2)$$

does not have a solution. Make an appropriate comment (with sound reasoning) about Jasper's knowledge. (Do not attempt to solve (2).)
5. a. Find a fundamental set of solutions for

\[ y'' - y' - 6y = 0. \]  \hspace{1cm} (3)

b. Find the particular solution of (1) satisfying the initial conditions

\[ y(0) = 3, \quad y'(0) = -1. \]

6. a. Prove that \( y_1 = t^2 \) and \( y_2 = 1/t \) form a fundamental set of solutions for the equation

\[ L[y] = t^2 y'' - 2y = 0. \]

b. Determine the largest interval(s) on which your fundamental set is valid.
7. Consider the initial value problem
\[ t(1 - t)y'' + 3y' - (2 - t)y = 0, \quad y(t_0) = y_0, \quad y'(t_0) = y'_0. \] (4)

a. What are the possible values of \( t_0 \).
b. Apply the basic existence and uniqueness theorem and indicate the intervals where there will exist a unique solution to (4).

8. Prove that the sequence \( f_n(t) = \frac{\cos(nt)}{\sqrt{n}}, n \to \infty \), is uniformly convergent on \([0, \infty)\).