Name February 15, 2019
Math 441 Exam No. 1

Why are wise few, fools numerous in the excesse,
'Cause, wanting number, they are numberlesse.

Noah Bridges: Vulgar (Common) Arithmetike, London, 1659.

SHOW ALL WORK. INDICATE ALL REASONING.

1. Find the general solution of the differential equation

$$y' - \frac{2}{t}y = t^5, \qquad t > 0.$$

2. Find the general solution of

$$(3x^2y^2 + x^2) + (2x^3y + y^2)y' = 0.$$

3. a. Find the general solution of

$$y' = \frac{t}{y^3(t^2-4)}, \qquad t > 0, \tag{1}$$

and determine where your solution is valid.

b. Find the particular solution of (1) satisfying the initial condition $y(\sqrt{5}) = 2$.

c. Determine the maximum value of h such that your solution is valid for

$$\sqrt{5} - h < \sqrt{5} < \sqrt{5} + h.$$

4. On his first exam in extraordinary differential equations, Jasper Figworthy, a student of dynamic engineering at Foghead State University in Old Hogwash Junction, claimed that the initial value problem

$$y' = \cos(y^2 e^{t^2}), \qquad y(0) = \pi^{13}, \tag{2}$$

does not have a solution. Make an appropriate comment (with sound reasoning) about Jasper's knowledge. (Do not attempt to solve (2).)

5. a. Find a fundamental set of solutions for

$$y'' - y' - 6y = 0. \qquad (3)$$

b. Find the particular solution of (1) satisfying the initial conditions

$$y(0) = 3, \qquad y'(0) = -1.$$

6. a. Prove that $y_1 = t^2$ and $y_2 = 1/t$ form a fundamental set of solutions for the equation

$$L[y] = t^2 y'' - 2y = 0.$$

b. Determine the largest interval(s) on which your fundamental set is valid.

7. Consider the initial value problem

$$t(1-t)y'' + 3y' - (2-t)y = 0, \qquad y(t_0) = y_0, \quad y'(t_0) = y'_0. \tag{4}$$

a. What are the possible values of t_0.

b. Apply the basic *existence and uniqueness theorem* and indicate the intervals where there will exist a unique solution to (4).

8. Prove that the sequence $f_n(t) = \dfrac{\cos(nt)}{\sqrt{n}}$, $n \to \infty$, is uniformly convergent on $[0, \infty)$.