1. Consider

\[ x^2y'' - 3xy' - 4(x - 1)y = 0. \]  

(1)

a. Determine the singular points of (1) and their types.
b. Find one solution of (1).
c. Where does your solution converge?
d. Write down the form of a second linearly independent solution of (1). (You do not have to find this second solution.)
Problem #1 Continued
2. Find the general solution of the system
\[ tX' = \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \mathbf{x}. \]
3. Consider the $n \times n$ system

$$X' = P(t)X,$$ (2)

where $P(t)$ is continuous on $(-\infty, \infty)$. Let $X^{(1)}, X^{(2)}, \ldots, X^{(n)}$ be a set of $n$ solutions of (2). Suppose that for some point $t_0$,

$$X^{(k)}(t_0) = \begin{bmatrix} 0 \\ \vdots \\ k \\ \vdots \\ 0 \end{bmatrix}, \quad 1 \leq k \leq n,$$

i.e., this initial vector has $k$ in the $k$th position and 0 in each of the other $n - 1$ positions. Prove that $X^{(1)}, X^{(2)}, \ldots, X^{(n)}$ is a fundamental set.
4. Using the method of Diagonalization, find a particular solution of the system

\[
X' = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t.
\]
Problem #4 Continued