Solutions, Homework Set # 7

Sect 4.2 # 27
12 y'''' + 31y'' + 75y''' + 37y' + 5y = 0

Char. poly. p(2) = 122 + 31k3 + 75k2 + 37k + 5 = 0

If a/b is a rational root, then a | 5 and b | 12. We observe that

-1/3 is a rational root, because, after simplification,

\[ p(-1/3) = \frac{364 - 364}{27} = 0. \]

By long division,

\[ p(2) = 122 + 27k2 + 15 = 3(4k3 + 9k2 + 22k + 5) \]

-1/3 is a rational root, because, after simplification,

\[ 4k3 + 9k2 + 22k + 5 = 4k2 + 8k + 20 = 4(k2 + 1k + 5) \]

The roots of \( k^2 + 2k + 5 \) are

\[ k = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm 2i. \]

Thus, the general solution is

\[ y(t) = c_1 e^{-t/3} + c_2 e^{-t/4} + c_3 e^{-t} \]

Sect 4.3 # 17
y'''' - 4y' - y'' + y = t^2 + 4 + 5 \sin t

Char. poly. \( n^4 - n^3 - n^2 + n = n(n^2 - n^2 - n + 4) = 0 \)

Note that \( n = 0, \pm 1 \) are roots.

\[ \frac{n^3 - n^2 - n + 4}{n - 1} = n^2 - 1 = 0 \Rightarrow n = \pm 1. \]

In conclusion, \( n = 0, \pm 1 \) are the characteristic roots

\( t^2 + 4 \) has degree 2; \( n = 0 \) has order 1. Thus,

\[ y_1(t) = t(A + B t + C) \]

\( t \) is a polynomial of degree 1; \( \pm i \) are not characteristic. Thus

\[ y_2(t) = t(\sin t + (D t + E) \cos t + (F t + G) \sin t) \]

\[ y_p(t) = t(A + B t + C) + (D t + E) \cos t + (F t + G) \sin t \]
Sect. 4.1, #34. Do three equal constants, not all 0, such that
\[
c_1 \sin^2 t + c_2 \sin^2 t + c_3 \cos(2t) = 0
\]
\[
5c_1 + (c_2 - c_3) \sin^2 t + c_3 \cos^2 t = 0
\]
\[
5c_1 + (c_2 - c_3) \sin^2 t + c_3 (1 - \sin^2 t) = 0
\]
\[
(5c_1 + c_2) + (c_2 - 2c_3) \sin^2 t = 0
\]
Thus,
\[5c_1 + c_3 = 0 \implies 10c_1 + 2c_3 = 0\]
\[c_2 - 2c_3 = 0\]
Add! \[10c_1 + c_2 = 0\].

We see that there are lots of values for \(c_1, c_2, c_3\) that are not all 0, e.g. \(c_1 = 1, c_2 = -10, c_3 = -5\). Thus, \(5, \sin^2 t, \) and \(\cos(2t)\) are linearly dependent.