Solutions, Homework Set #7

Sect. 4.2, #27 $12y^{(4)}+31y'''+75y''+37y'+5y=0$

Char. poly. $p(r)=12r^4+31r^3+75r^2+37r+5=0$.

If a/b is a rational root, then $a|5$ and $b|12$. We observe that $-1/3$ is a rational root, because, after simplification,

$$p(-1/3)=\frac{364}{27}-\frac{364}{27}=0.$$

By long division,

$$\frac{p(r)}{r+1/3}=12r^3+27r^2+66r+15=3(4r^3+9r^2+22r+5)$$

$-1/4$ is a rational root, because, after simplification

$$\frac{4r^3+9r^2+22r+5}{r+1/4}=4r^2+8r+20=4(r^2+2r+5)$$

The roots of r^2+2r+5 are

$$r=\frac{-2\pm\sqrt{4-20}}{2}=-1\pm 2i.$$

Thus, the general solution is

$$y(t)=c_1e^{-t/3}+c_2e^{-t/4}+c_3e^{-t}\cos(2t)+c_4e^{-t}\sin(2t)$$

Sect 4.3, #17 $y^{(4)}-y'''-y''+y'=t^2+4+t\sin t$

Char. poly. $r^4-r^3-r^2+r=r(r^3-r^2-r+1)=0$

Note that $r=0,1$ are roots.

$$\frac{r^3-r^2-r+1}{r-1}=r^2-1=0\Rightarrow r=\pm 1$$

In conclusion, $r=0,1,1,-1$ are the characteristic roots

t^2+4 has degree 2; $r=0$ has order 1. Thus,

$$y_{p1}(t)=t(At^2+Bt+C)$$

t is a polynomial of degree 1; $\pm i$ are not char. roots. Thus

$$y_{p2}(t)=(Dt+E)\cos t+(Ft+G)\sin t$$

$$\therefore\ y_p(t)=t(At^2+Bt+C)+(Dt+E)\cos t+(Ft+G)\sin t$$

Math 441, Solutions Homework Set #7

Sect. 4.1, #17 Do there exist constants, not all 0, such that

$$c_1 \cdot 5 + c_2 \sin^2 t + c_3 \cos(2t) = 0$$

or $$c_1 \cdot 5 + c_2 \sin^2 t + c_3 (\cos^2 t - \sin^2 t) = 0$$

or $$5c_1 + (c_2 - c_3)\sin^2 t + c_3 \cos^2 t = 0$$

or $$5c_1 + (c_2 - c_3)\sin^2 t + c_3(1 - \sin^2 t) = 0$$

or $$(5c_1 + c_3) + (c_2 - 2c_3)\sin^2 t = 0$$

Thus, $$5c_1 + c_3 = 0 \Rightarrow 10c_1 + 2c_3 = 0$$
$$c_2 - 2c_3 = 0$$

Add: $$10c_1 + c_2 = 0.$$

We see that there are lots of values for c_1, c_2, c_3 that are not all 0, e.g. $c_1 = 1, c_2 = -10, c_3 = -5$. Thus, 5, $\sin^2 t$, and $\cos(2t)$ are linearly dependent.