Sec 3.1, #23 \[ y'' - (2x-1)y' + x(x-1)y = 0 \]
\[ 2x^2 - (2x-1)x + x(x-1) = (2x-1)(x-1) = 0 \]
Characteristic roots are \(x, x-1\). Thus, a general solution is
\[ y = C_1 e^{x} + C_2 e^{(x-1)} \]
1. \(x > 1\), then \(e^{x}, e^{(x-1)} \to \infty\) as \(t \to \infty\).
2. \(x < 0\), then \(e^{x}, e^{(x-1)} \to 0\) as \(t \to \infty\).

Sec 3.2, #25 \[ y'' - 2y' + y = 0 \]
\[ 2x^2 - 2x + 1 = (2x-1)^2 = 0 \]
Characteristic roots are \(1, 1\). Thus, \(y_1(t) = e^t\) is a solution.
Let \(y_2(t) = te^t\). \(y_2'(t) = te^t + e^t, y_2''(t) = te^t + 2e^t\)
\[ L(y_2) = te^t + 2e^t - 2(te^t + e^t) + te^t = 0 \]
Thus, \(y_2\) is a solution.

\[ W(y_1, y_2) = \begin{vmatrix} e^t & te^t \\ e^t & te^t + e^t \end{vmatrix} = te^t + e^{2t} - te^t = e^{2t} \neq 0 \]
Thus, \(e^t, te^t\) form a fundamental set.

Sec 3.2, #27 \[ L(x) = (1-x \cot x) \cdot 0 - x. 1 + x = 0 \quad \therefore \quad x \text{ is a solution} \]

\[ L(\sin x) = (1-x \cot x)(-\sin x) - x \cos x + \sin x \]
\[ = -\sin x + x \cos x - x \cos x + \sin x = 0 \quad \therefore \quad \sin x \text{ is a solution} \]

\[ W(x, \sin x) = \begin{vmatrix} x & \sin x \\ 1 & \cos x \end{vmatrix} = x \cos x - \sin x \]
At \(x = \frac{\pi}{2}\), \(W(x, \sin x) = 0 - 1 = -1 \neq 0 \)
Thus, \(x, \sin x\) form a fundamental set.