Math 441, Solutions, Homework Set #5

Sect. 3.1, #23 $y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0$

$$r^2 - (2\alpha - 1)r + \alpha(\alpha - 1) = (r - \alpha)(r - (\alpha - 1)) = 0$$

characteristic roots are $\alpha, \alpha - 1$. Thus, a general solution is

$$y = c_1 e^{\alpha t} + c_2 e^{(\alpha - 1)t}$$

If $\alpha > 1$, then $e^{\alpha t}, e^{(\alpha-1)t} \to \infty$ as $t \to \infty$.
If $\alpha < 0$, then $e^{\alpha t}, e^{(\alpha-1)t} \to 0$ as $t \to \infty$.

Sect 3.2, #25 $y'' - 2y' + y = 0$

$$r^2 - 2r + 1 = (r-1)^2 = 0$$

characteristic roots are $1, 1$. Thus, $y_1(t) = e^t$ is a solution.
Let $y_2(t) = te^t$. $y_2'(t) = te^t + e^t$, $y_2''(t) = te^t + e^t + e^t$
$= te^t + 2e^t$

$L(y_2) = te^t + 2e^t - 2(te^t + e^t) + te^t = 0$.
Thus, y_2 is a solution.

$$W(y_1, y_2) = \begin{vmatrix} e^t & te^t \\ e^t & te^t + e^t \end{vmatrix} = te^{2t} + e^{2t} - te^{2t} = e^{2t} \neq 0$$

Thus, e^t, te^t form a fundamental set.

Sect. 3.2 #29 We use Abel's theorem on page 154. Write the d.e. in the form

$$y'' - \frac{t+2}{t}y' + \frac{t+2}{t^2}y = 0$$

Here $p(t) = -\frac{t+2}{t} = -1 - \frac{2}{t}$. By Abel's theorem,

$$W(y_1, y_2) = c\exp\left(-\int\left(-1 - \frac{2}{t}\right)dt\right) = c\exp\int\left(1 + \frac{2}{t}\right)dt$$

$$= c\exp(t + 2\log t) = ce^t e^{2\log t} = ce^t t^2$$