Math 441, Solutions, HW Set # 2

2.6, # 23 In class, we showed that if $\mu(x,y)$ is an integrating factor for $M+Ny'=0$, then

$$M\mu_y - N\mu_x = \mu(N_x - M_y).$$

If μ is a function of y only, then the equation above reduces to

$$M\mu_y = \mu(N_x - M_y)$$

or

$$\frac{\mu_y}{\mu} = \frac{N_x - M_y}{M} \Rightarrow \log \mu(y) = \int \frac{N_x - M_y}{M}\,dy$$

Thus,

$$\mu(y) = \exp\left(\int \frac{N_x - M_y}{M}\,dy\right)$$

2.6 #27 $1 + \left(\frac{x}{y} - \sin y\right)y' = 0$

$M_y = 0,\ N_x = 1/y \quad \therefore$ not exact

We try to find an integrating factor $\mu(y)$. We need

$$\frac{N_x - M_y}{M} = \frac{1/y - 0}{1} = \frac{1}{y}$$, indeed a function of y only

$\therefore$

$$\mu(y) = e^{\int \frac{dy}{y}} = e^{\log y} = y$$

$\therefore\ y + (x - y\sin y)y' = 0 \qquad (*)$

Thus, as $(*)$ is now exact, $\exists$ a function $\Psi(x,y) \ni$:

$$\Psi_x(x,y) = y, \quad \Psi_y = x - y\sin y$$

$$\therefore\ \Psi(x,y) = xy + h(y)$$

$$\Psi_y = x + h'(y) = x - y\sin y$$

$$\Rightarrow \quad h'(y) = -y\sin y \quad \cancel{c_1,\ c_1 \text{ constant}}$$

$$\Rightarrow \quad h(y) = -\int y\sin y\,dy + c_1, \quad c_1 \text{ constant}$$

$$= y\cos y - \int \cos y\,dy + c_1 = y\cos y - \sin y + c_1$$

$$\therefore\ \Psi(x,y) = xy + y\cos y - \sin y + c_1$$

$$\therefore\ xy + y\cos y - \sin y = C$$