Math 441, Solutions HW Set #1

2.1 # 17 $y' - 2y = e^{2t}$, linear 1st order

integrating factor is $e^{\int -2dt} = e^{-2t}$

$\therefore \frac{d}{dt}\left(e^{-2t} y(t)\right) = e^{2t} \cdot e^{-2t} = 1$

$\therefore \quad e^{-2t} y(t) = t + c$

$y(0) = 2 \Rightarrow e^{-0} \cdot 2 = 0 + c \Rightarrow c = 2$

$\therefore y(t) = e^{2t}(t+2)$

2.1 # 39 $A(t) = \int e^{\int -2dt} t^2 e^{2t} dt + c$

$= \int e^{-2t} t^2 e^{2t} dt + c = \int t^2 dt + c = \frac{t^3}{3} + c$

$\therefore y(t) = e^{\int 2dt} A(t) = e^{2t}\left(\frac{t^3}{3} + c\right)$

2.1 # 41

$A(t) = \int e^{\int \frac{2}{t} dt} \frac{\sin t}{t} dt + c$

$= \int e^{2 \log t} \frac{\sin t}{t} dt + c = \int t^2 \frac{\sin t}{t} dt + c$

$= \int t \sin t \, dt + c$

$= -t\cos t + \int \cos t \, dt + c$

$= -t\cos t + \sin t + c$

$\therefore y(t) = e^{\int -\frac{2}{t} dt}(-t\cos t + \sin t + c)$

$= t^{-2}(-t\cos t + \sin t + c)$

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2.2 #23 $y' = 2y^2 + xy^2 = (2+x)y^2$ is variables separable

$\therefore\ y^{-2}dy = (2+x)dx \Rightarrow \dfrac{y^{-1}}{-1} = 2x + \dfrac{x^2}{2} + C$

$y(0) = 1 \Rightarrow -1 = 0 + C \Rightarrow C = -1$

$\therefore\ \dfrac{y^{-1}}{-1} = 2x + \dfrac{x^2}{2} - 1$

or $y = \dfrac{-1}{2x + x^2/2 - 1}$

$y' = \dfrac{2+x}{(2x + x^2/2 - 1)^2} = 0$ if $x = -2$

$y(-2) = \dfrac{-1}{-4+2-1} = \dfrac{-1}{-3} = \dfrac{1}{3}$

Note: $y(0) = 1$, thus $1/3$ is a local minimum and not a local maximum.

2.2 #32 $y' = \dfrac{x^2+3y^2}{2xy} = \dfrac{x}{2y} + \dfrac{3y}{2x}$

Let $u = y/x$, or $ux = y \Rightarrow y' = u'x + u$

Thus, $u'x + u = \dfrac{1}{2u} + \dfrac{3u}{2} \Rightarrow u'x = \dfrac{1}{2u} + \dfrac{u}{2} = \dfrac{u^2+1}{2u}$

$\therefore\ \dfrac{2u\,du}{u^2+1} = \dfrac{dx}{x}$

$\displaystyle\int \frac{2u\,du}{u^2+1} = \int \frac{dx}{x} + \log C$

$\log(u^2+1) = \log x + \log C = \log(xC)$

$\therefore\ u^2 + 1 = xC$

$\dfrac{y^2}{x^2} + 1 = xC$

$y^2 + x^2 = x^3 C$