Abstract:
In this talk, we describe some interactions between the combinatorics and algebraic geometry of the following setting studied by A. Buch and W. Fulton: Let $X$ be a smooth complex algebraic variety and $E_0 \to E_1 \to \cdots \to E_n$ be a sequence of vector bundles and maps over $X$. This gives rise to a “degeneracy locus” in $X$. What are formulas for this locus in the $K$-theory? This question was originally answered in terms of combinatorially defined “Quiver coefficients”, which were conjectured to alternate in sign according to codimension. Combinatorial formulas for the Quiver coefficients, especially those that explain the alternating signs, are of interest since they provide, e.g.,:

1. new “Giambelli-type” formulas for Schubert classes, for both classical and quantum cohomology of partial flag varieties;
2. generalizations of the classical and K-theory Littlewood-Richardson rules for Grassmannians;
3. generalizations of the classical Giambelli-Thom-Porteous determinantal formulas.

We present explanations for the alternating sign phenomenon and give new combinatorial formulas for the Quiver coefficients. Also, we suggest a geometric rationale for the alternating signs: the Quiver coefficients are Schubert structure constants for flag varieties. Combinatorial generalizations of the Quiver coefficients to the other classical Lie types will also be discussed.

Our answers will involve semistandard tableaux, reduced words of permutations and the combinatorics of Schubert polynomials.

This talk is based on math.AG/0211300, math.CO/0306389, math.CO/0307019 and math.CO/0311390.

Host: J. Remmel

Tuesday, December 2, 2003
4:00 pm
AP&M 7421