Answer as many problems as you can. Each question is worth 6 points (total points is 30). Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page. [Total time: 50 minutes]

Below are some formulas you might find helpful. This list is neither guaranteed to be complete, nor do I guarantee that each formula is actually needed on the exam.

\[
{n \choose k} = {n - 1 \choose k} + {n - 1 \choose k - 1}
\]

\[(x + y)^n = \sum_{j=0}^{n} {n \choose k} x^j y^{n-j}, n \in \mathbb{N}\]

\[
|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum_{i_1} |A_{i_1}| - \sum_{i_1, i_2} |A_{i_1} \cap A_{i_2}| + \cdots + (-1)^{n+1} \sum_{i_1, \ldots, i_n} |A_{i_1} \cap \cdots \cap A_{i_n}|
\]

\[
|A_1^c \cap A_2^c \cap \cdots \cap A_n^c| = |S| - \sum_{i_1} |A_{i_1}| + \sum_{i_1, i_2} |A_{i_1} \cap A_{i_2}| - \cdots + (-1)^n \sum_{i_1, \ldots, i_n} |A_{i_1} \cap \cdots \cap A_{i_n}|
\]

See the back page for addition study notes and pointers
1. Use the binomial theorem and the fact that
\[(1 + x)^a(1 + x)^b = (1 + x)^{a+b}\]
to prove the identity
\[\sum_{k=0}^{n} \binom{a}{k} \binom{b}{n-k} = \binom{a+b}{n}.\]
(Do not give a combinatorial proof.)
2. Prove that every positive integer has a (positive integer) multiple whose digits are all 0’s or 1’s.
3. Prove that if you are given \( n^2 + 1 \) distinct real numbers there is an increasing subsequence of length \( n + 1 \) or a decreasing subsequence of length \( n + 1 \).
4. Prove that in a group of $n > 1$ people there are two who have the same number of acquaintances in the group. (It is assumed that no one is acquainted with oneself.)
5. Prove that if \( i < \lfloor n/2 \rfloor \) then \( \binom{n}{i} \leq \binom{n}{i+1} \).
(This page is intentionally left blank).
Midterm 2 will be based on the material from Lecture 7 (the Pigeonhole principle) to Lecture 13 (which includes the proof of the general inclusion exclusion formula). It will also cover homeworks 3, 4 and 5.

Here are the major theorems I consider highly testable

• Erdös-Szekeres Theorem (lecture 8)
• The six person party problem (lecture 9)
• Ramsey’s theorem (that $R(p, q)$ exists) (lecture 9)
• The binomial theorem (lecture 10)
• Unimodality of the binomial coefficients (lecture 11)
• Sperner’s theorem (lecture 12)
• Proof of the complementary form of the inclusion-exclusion formula (lecture 13)
• Formula for $\sum_{k=1}^{n} k^j$ ($j$ fixed) as discussed in the worksheet and in class.

Where applicable I may ask for the statement and the proof. Since I’m telling you in advance, I expect the proofs to be well-done in these cases. I could also, in some cases ask you for parts of a proof.

All of the material in the lectures, assignments, worksheets, and even this practice test are testable. However, I will emphasize things discussed at length in class. So for example, I’m far more likely to ask about a problem from the notes we talked about a lot than one that we skipped in class. That said, you might wish to go over everything as practice.

Pigeonhole principle problems: It’s easy to find many examples, e.g., from the textbook or the internet. I’ll probably pick one like that myself.

Homework problems: I’ll likely ask a problem straight from your homework, or a slight variation.

Good luck and happy studying!