U. of Illinois MATH 413 Chapter 6 In class exercises

1. How many ways are there to select a 5-card hand from a regular 52 card deck such that the hand contains at least one card in each suit?
2. How many ways are there to arrange the letters in INTELLIGENT with at least two pairs of consecutive identical letters? (For example ITTNELLGENI is an arrangement we want to count, since it has “TT” and “LL”.)
3. I have five friends. During a long holiday, I met any given friend at dinner 10 times, and any given pair of friends 5 times, and given threesome of friends 3 times and any given foursome of friends 2 times, and all friends at once 1 time. If I ate alone 6 times, how long was my holiday?
   (In this problem, meeting “any given pair (or threesome or foursome) of friends” means I meet those two friends and possibly some others during certain days.)
4. This problem is more practice with the ideas in Section 6.4: Consider the 5x5 grid:
   
   \[
   \begin{array}{ccccc}
   X & \cdot & \cdot & X & \cdot \\
   \cdot & \cdot & X & \cdot & \cdot \\
   \cdot & \cdot & \cdot & \cdot & \cdot \\
   X & \cdot & \cdot & X \\
   X & \cdot & \cdot & X \\
   \end{array}
   \]
   
   where the “X”’s denote “forbidden squares”. Your goal is to count the number of non attacking rook placements (5 rooks) into the 5x5 grid such that no rook uses any of the forbidden squares.
   (a) What are the sets A1,...,A5?
   (b) Use the complementary form of the inclusion exclusion formula to derive the answer.
   (c) Can you generalize your answer to (b) for arbitrary nxn grids with given forbidden squares?
Brief hints:

0. The (complementary form of the) inclusion-exclusion principle says

\[ |A_1^c \cap \cdots \cap A_m^c| = |S| - \sum |A_i| + \sum |A_i \cap A_j| \pm \cdots \]

1. \( C(52, 5) - 4C(39, 5) + 6C(26, 5) - 4C(13, 5). \)

2. \[ [C(5, 2) \times 9!/(2!)^3] - [2 \times C(5, 3) \times 8!/(2!)^2] + [3 \times C(5, 4) \times 7!/2!] - [4 \times 6!] \].

3. 27

4. Answer to (b): 5! - (7\times 4!) + (16\times 3!) - 14\times 2! + 4\times 1!.