Ch. 3 Insurance, collars, and other strategies

One can insure a long position in a stock by buying a put, establishing a floor on the sales price of the stock.

Purchase price of stock = $S_0$ (at time $t=0$)
Expiration date = $T$, risk-free interest rate = $r$, interest is compounded continuously (we assume this)
strike price (of put) = $K$, put premium paid for the put = $P$

For the combined position:

payoff (at time $T$) = $S_T + (K-S_T)^+$ = \[
\begin{cases}
S_T - (S_T - K) & \text{if } S_T > K \\
K & \text{if } S_T \leq K
\end{cases}
\]

profit (at time $T$) = $S_T + (K-S_T)^+ - (S_0 + P) e^{rT}$
= \[
\begin{cases}
S_T - (S_0 + P) e^{rT} & \text{if } S_T > K \\
K - (S_0 + P) e^{rT} & \text{if } S_T \leq K
\end{cases}
\]

Same general shape as for a long call option.

payoff:

profit:
One can insure a short position in a stock by buying a call, establishing a cap on the potential loss.

Notation as above. Now $K =$ strike price of the call, stock shorted at price $S_0$, premium paid for call = $p$.

For the combined position

Payoff (at time $T$) $= -S_T + (S_T - K)^+$

$= \begin{cases} 
-S_T, & S_T \leq K \\
-K, & S_T > K 
\end{cases}$

Profit (at time $T$) $= -S_T + (S_T - K)^+ + (S_0 - p)e^{rT}$

$= \begin{cases} 
(S_0 - p)e^{rT} - S_T, & S_T \leq K \\
(S_0 - p)e^{rT} - K, & S_T > K 
\end{cases}$

Same general shape as for a long put.

Def: Writing options = selling options
Covered call: Buying a stock and selling a call.

Naked call: Selling a call (not owning the stock)

Naked put: Selling a put (not being short the stock)

Covered put: Shorting the stock and selling a put

Payoff and profit diagrams: exercise
Option spreads

Vertical spreads

Buying and selling options (both calls or both puts) for the same stock, for the same expiration date, at different strike prices.

Difference in strike prices

Bull spread: based on the belief that the stock price will go up

Bull call: buy call at strike price \( K_1 \)

sell \( K_2 > K_1 \)

pay a premium, but less than if only buying a call

Max payoff: \( K_2 - K_1 \)

Max profit: \( K_2 - K_1 - Pe^{rt} \)

Max loss: \(-Pe^{rt}\) (abs. value of max. loss = \(Pe^{rt}\))

Bull put: buy put at \( K_1 \)

sell put at \( K_2 > K_1 \)

Receive premium \( P \)

Max payoff: \( 0 \)

Max profit: \( Pe^{rt} \)

Max loss: \( Pe^{rt}(K_2 - K_1) \)

(abs. value of max. loss = \((K_2 - K_1) - Pe^{rt}\))
Bear spreads: based on the belief that the stock price will go down

Bear call: buy call at $K_1$
    sell call at $K_2 < K_1$
    receive premium $P$
    Max. payoff: 0
    Max. profit: $P e^{-T}$
    Max. loss: $P e^{-T} (K_1 - K_2) < 0$

Bear put: buy put at $K_1$
    sell put at $K_2 < K_1$
    pay $P$
    Max. payoff: $K_1 - K_2$
    Max. profit: $(K_1 - K_2) - P e^{-T}$
    Max. loss: $-P e^{-T}$

Payoff and profit diagrams: exercise

Example: Bull put

Payoff

Profit

$P' = P e^{r T}$

$P' (K_2 - K_1) < 0$
Box spread

Buy call and sell put at strike price \( K_1 \),
sell call and buy put at \( K_2 > K_1 \),

Payoff = \( K_2 - K_1 \), no matter what \( S_T \) is.
Possible reasons for use: taxes, regulations.
Most pay premium for this.
Premium should be \(< (K_2 - K_1)\), but
might not be, e.g., due to
- \( T \) small
- \( r \) small
- bid-ask spread not small

Use SPX as an example.

Ex. (generic stock) (not SPX)
- buy a call and sell a put at 40
- sell a call and buy a put at 45
Payoff = 5
Theoretical cost a bit under 5, but in practice
cost may be \( > 5 \), making this deal impractical.
Ratio spread

Buy \( m \) options at strike price \( K_1 \),
sell \( n \) at strike price \( K_2 \),
\((m+n, K_1 + K_2)\)

Properties depend on \( m, n, K_1, K_2 \).
Option involved can be call or put.
Collar

Purchased (long) collar:
Buy a put at \( K_1 \)
sell a call at \( K_2 \) \( (K_2 > K_1) \).

Collar width = \( K_2 - K_1 \).

Premium paid or received, depending on \( K_1 \) and \( K_2 \). Premium = 0 = zero-cost collar

Profit diagram:

\[
\begin{array}{c}
K_1 \quad K_2 \\
\end{array}
\]

Sold (short) collar: opposite of the above
sell put \( K_1 \)
buy call \( K_2 > K_1 \).

Collared stock: buy the stock, sell a call at strike price \( K_1 \), buy a put a strike price \( K_2 \), where \( K_2 < S_0 < K_1 \).
Straddle

Buy a call and buy a put at same strike price $K$.

Strangle

Buy a put at $K_1$, Buy a call at $K_2$ ($K_1 < S < K_2$)

Both based on hope of high volatility; a large movement in stock price in either direction.

Used e.g. before earnings.

Profit

Straddle Strangle
Butterfly spreads

Butterfly = combination of two vertical spreads in opposite directions (one bull, one bear)

E.g.,
Buy a call at \( K \) and \( K+2\varepsilon \)
Sell 2 calls at \( K+\varepsilon \)

Net premium paid:

\[
\frac{\text{Max profit when } S_T = K + \varepsilon}{\text{Max loss: } -P e^{-rT}} = \varepsilon - P e^{-rT}
\]

Profit:

One can also sell this butterfly.

Similarly, one can buy or sell a butterfly consisting of two put spreads or one call and one put spread (iron condor).
Ch. 4 Introduction to risk management

Producer's perspective
E.g. Hedging with a forward contract
Wants to sell the product at a future time

Possibilities
- do nothing before the sale (unhedged position)
- short a forward contract
- buy a put
- buy a put and sell a call

Choice of strike prices is important.
Final result depends on price at future time,
and different choices do better under different prices at that time.
Buyer's perspective

Wants to buy a product at a future time.

Possibilities:
- do nothing before purchase (unhedged position)
- long forward contract
- buy a call